

Likelihood Ratios for Mixtures: Semi-continuous Approach

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Acknowledgement

I thank Michael Coble, Bruce Weir and John Buckleton for their helpful discussions.

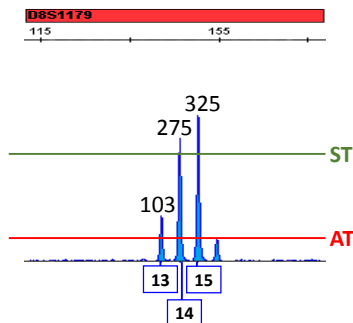
Disclaimer

Points of view in this presentation are mine and do not necessarily represent the official position or policies of the National Institute of Standards and Technology.

Overview of different approaches

		Takes into account:		Models:	
		presence/absence of alleles	possible genotypes based on peak heights	allele drop-out and allele drop-in	peak heights
Binary	CPI	X			
	mRMP	X	X		
	LR (binary)	X	X		
Probabilistic genotyping	LR (semi-continuous)	X		X	
	LR (fully continuous)	X	X	X	X

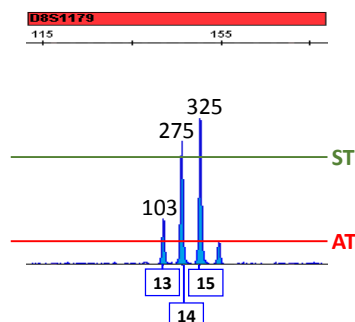
Binary LR



$$G_{POI} = \{13, 16\}$$

numerator of LR: $1 \times 2p_{14}p_{15}$

Binary LR



$$G_{POI} = \{13,16\}$$

denominator of LR (unrestricted): $12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q$

denominator of LR (restricted): $2p_{14}p_{15}p_{13}(p_{13} + 2p_{14} + 2p_{15} + 2p_Q)$

Binary LR

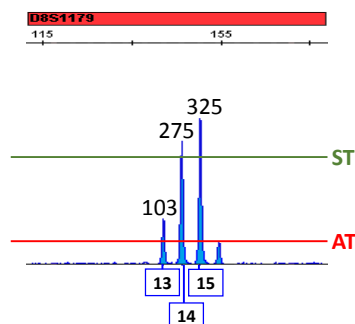
unrestricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q}$$

restricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{2p_{14}p_{15}p_{13}(p_{13} + 2p_{14} + 2p_{15} + 2p_Q)}$$

Binary LR



$$G_{POI} = \{13, 13\}$$

Binary LR

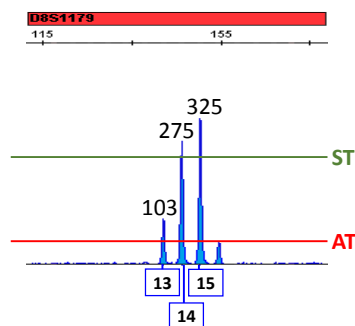
unrestricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q}$$

restricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{2p_{14}p_{15}p_{13}(p_{13} + 2p_{14} + 2p_{15} + 2p_Q)}$$

Binary LR



$$G_{POI} = \{12, 13\}$$

Binary LR

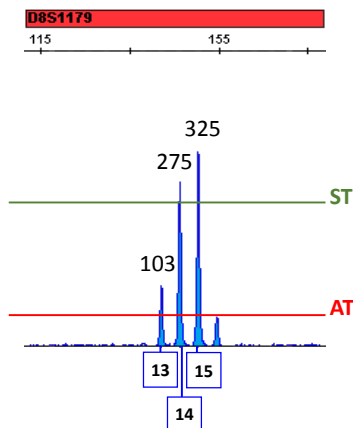
unrestricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q}$$

restricted:

$$LR = \frac{1 \times 2p_{14}p_{15}}{2p_{14}p_{15}p_{13}(p_{13} + 2p_{14} + 2p_{15} + 2p_Q)}$$

Binary LR



Shouldn't there be a difference in the value of the evidence for each of these POIs depending on how probable allele drop-out is?

Likelihood Ratio

$$\frac{\Pr(E|H_p, I)}{\Pr(E|H_d, I)}$$

E : DNA typing results

G_{CS} : crime stain

G_K : known contributors

$\left\{ \begin{array}{l} G_V: \text{victim or complainant} \\ G_{POI}: \text{person of interest} \end{array} \right.$

Likelihood Ratio

$$\begin{aligned}
 LR &= \frac{\Pr(E|H_p, I)}{\Pr(E|H_d, I)} \\
 &= \frac{\Pr(G_{CS}, G_K|H_p, I)}{\Pr(G_{CS}, G_K|H_d, I)} \\
 &= \frac{\Pr(G_{CS}|G_K, H_p, I)}{\Pr(G_{CS}|G_K, H_d, I)} \times \frac{\Pr(G_K|H_p, I)}{\Pr(G_K|H_d, I)} \\
 &= \frac{\Pr(G_{CS}|G_K, H_p, I)}{\Pr(G_{CS}|G_K, H_d, I)} \times \underbrace{\frac{\Pr(G_K|I)}{\Pr(G_K|I)}}_1
 \end{aligned}$$

The genotypes of the POI and victim/complainant do not depend on H_p being true or H_d being true.

Likelihood Ratio

The probability of obtaining these DNA typing results for the crime stain given the genotype of the POI (and victim/complainant) and that the POI is a contributor.

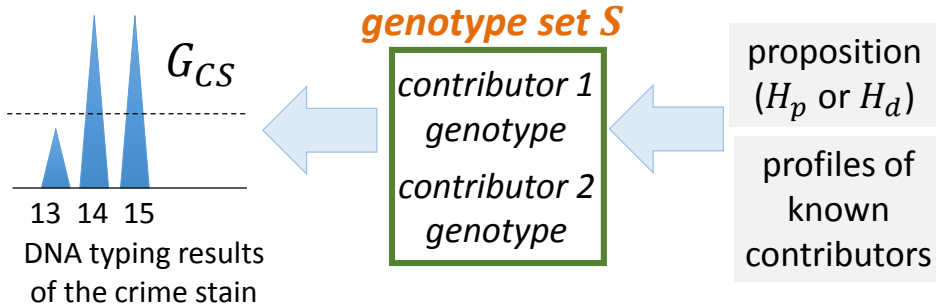
$$LR = \frac{\Pr(G_{CS}|G_K, H_p)}{\Pr(G_{CS}|G_K, H_d)}$$

The probability of obtaining these DNA typing results for the crime stain given the genotype of the POI (and victim/complainant) and that the POI is not a contributor.

Note that the background information I has been omitted in the above equation to focus your attention on G_{CS}, G_K, H_p and H_d .

Likelihood Ratio

The probability of G_{CS} depends on the genotypes considered for the contributors.



Genotype set S

binary

	Major	Minor	w	$match\ prob_{-major}$	$match\ prob_{-minor}$
S_1	14,15	12,13	1	$2p_{14}p_{15}$	$2p_{12}p_{13}$
S_2	14,15	13,13	1	$2p_{14}p_{15}$	p_{13}^2
S_3	14,15	13,14	1	$2p_{14}p_{15}$	$2p_{13}p_{14}$
S_4	14,15	13,15	1	$2p_{14}p_{15}$	$2p_{13}p_{15}$
S_5	14,15	13,16	1	$2p_{14}p_{15}$	$2p_{13}p_{16}$
S_6	14,15	13,17	1	$2p_{14}p_{15}$	$2p_{13}p_{17}$

Likelihood Ratio

$$\begin{aligned}
 LR &= \frac{\Pr(G_{CS}|G_K, H_p)}{\Pr(G_{CS}|G_K, H_d)} \\
 &= \frac{\Pr(G_{CS}|S_1)\Pr(S_1|G_K, H_p) + \Pr(G_{CS}|S_2)\Pr(S_2|G_K, H_p) + \dots + \Pr(G_{CS}|S_6)\Pr(S_6|G_K, H_p)}{\Pr(G_{CS}|S_1)\Pr(S_1|G_K, H_d) + \Pr(G_{CS}|S_2)\Pr(S_2|G_K, H_d) + \dots + \Pr(G_{CS}|S_6)\Pr(S_6|G_K, H_d)} \\
 &= \frac{\sum_{j=1}^6 \Pr(G_{CS}|S_j)\Pr(S_j|G_K, H_p)}{\sum_{j=1}^6 \Pr(G_{CS}|S_j)\Pr(S_j|G_K, H_d)}
 \end{aligned}$$

Note that the background information I has been omitted in the above equation to focus your attention on S_j, G_{CS}, G_K, H_p and H_d .

Likelihood Ratio

$$LR = \frac{\sum_{j=1}^m \Pr(G_{CS}|S_j)\Pr(S_j|G_K, H_p, I)}{\sum_{j=1}^m \Pr(G_{CS}|S_j)\Pr(S_j|G_K, H_d, I)}$$

weights

the probability of obtaining these DNA typing results for genotype set S_j

Likelihood Ratio

$$LR = \frac{\sum_{j=1}^m \Pr(G_{CS}|S_j) \Pr(S_j|G_K, H_p, I)}{\sum_{j=1}^m \Pr(G_{CS}|S_j) \Pr(S_j|G_K, H_d, I)}$$

match probabilities

the probability of genotype set S_j given that we have observed genotypes G_K and that the contributors are as specified in H_p or H_d

Likelihood Ratio:
from binary to semi-continuous

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

stays the same

(Hardy-Weinberg Law,
NRC II recommendation 4.1,
NRC II recommendation 4.2)

Likelihood Ratio: from binary to semi-continuous

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

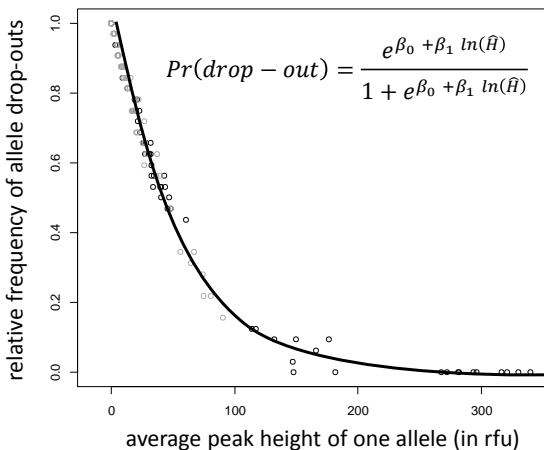
changes

binary weight: value of 0 (impossible) or 1 (possible)

semi-continuous weight: any value between 0 and 1, including 0 and 1, assigned using probabilities of drop-out and drop-in

Semi-Continuous LR

probability of allele drop-out



Semi-Continuous LR

probability of allele drop-out

heterozygote: $Pr(drop - out) = D$

$$Pr(not\ drop - out) = 1 - D = \bar{D}$$

homozygote: $Pr(drop - out) = D_2$

$$Pr(not\ drop - out) = 1 - D_2 = \bar{D}_2$$

based on the notation in:

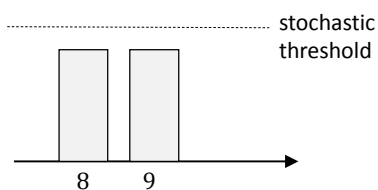
J. Buckleton, C.M. Triggs, S.J. Walsh. (2005). *Forensic DNA Evidence Interpretation*. CRC Press, London.

Semi-Continuous LR

Examples:



{8,9}



$$LR = \frac{\bar{D}^2}{\bar{D}^2 \times 2p_8p_9} = \frac{1}{2p_8p_9}$$

$$Pr(G_{CS}|G_K, H_p, I) = \bar{D} \bar{D} \times 1 = \bar{D}^2$$

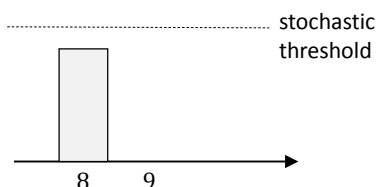
$$Pr(G_{CS}|G_K, H_d, I) = \bar{D} \bar{D} \times 2p_8p_9 = \bar{D}^2 \times 2p_8p_9$$

Semi-Continuous LR

Examples:



{8,9}



Contributor	
8,8	$\bar{D}_2 \times p_8^2$
8,Q	$\bar{D} D \times 2p_8p_Q$

where $p_Q = 1 - p_8$

$$Pr(G_{CS}|G_K, H_p, I) = \bar{D} D \times 1 = \bar{D} D$$

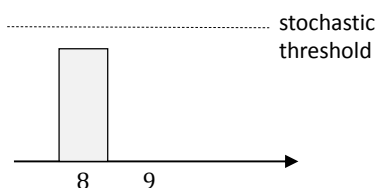
$$Pr(G_{CS}|G_K, H_d, I) = \bar{D}_2 \times p_8^2 + \bar{D} D \times 2p_8p_Q$$

Semi-Continuous LR

Examples:



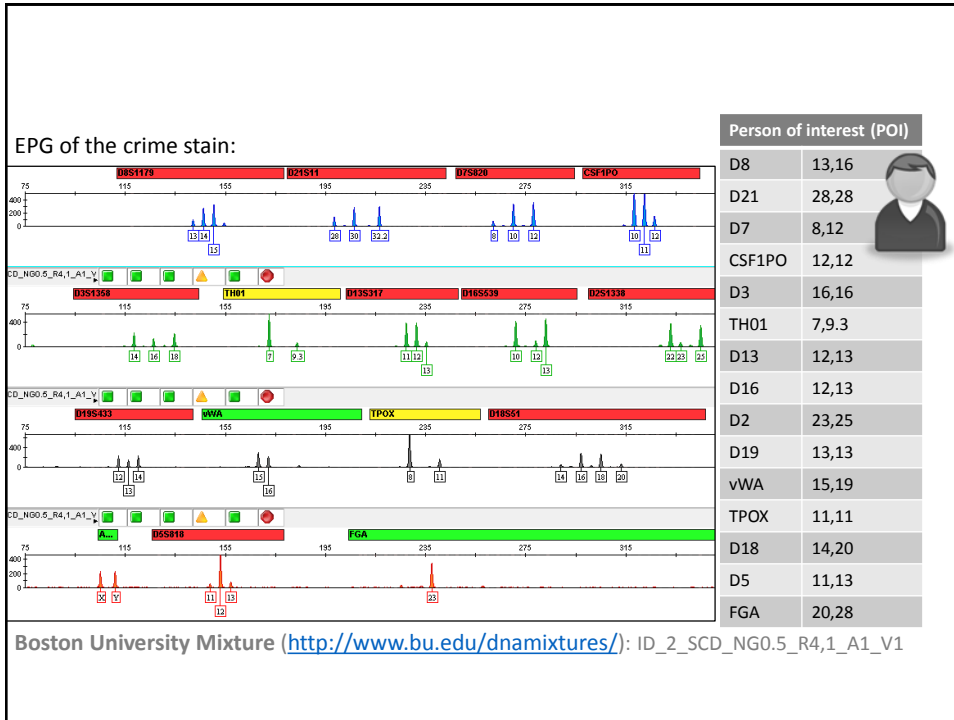
{8,9}



$$LR = \frac{\bar{D} D}{\bar{D}_2 \times p_8^2 + \bar{D} D \times 2p_8p_Q}$$

$$Pr(G_{CS}|G_K, H_p, I) = \bar{D} D \times 1 = \bar{D} D$$

$$Pr(G_{CS}|G_K, H_d, I) = \bar{D}_2 \times p_8^2 + \bar{D} D \times 2p_8p_Q$$



Assumptions

The DNA of the crime stain is a mixture of two contributors.

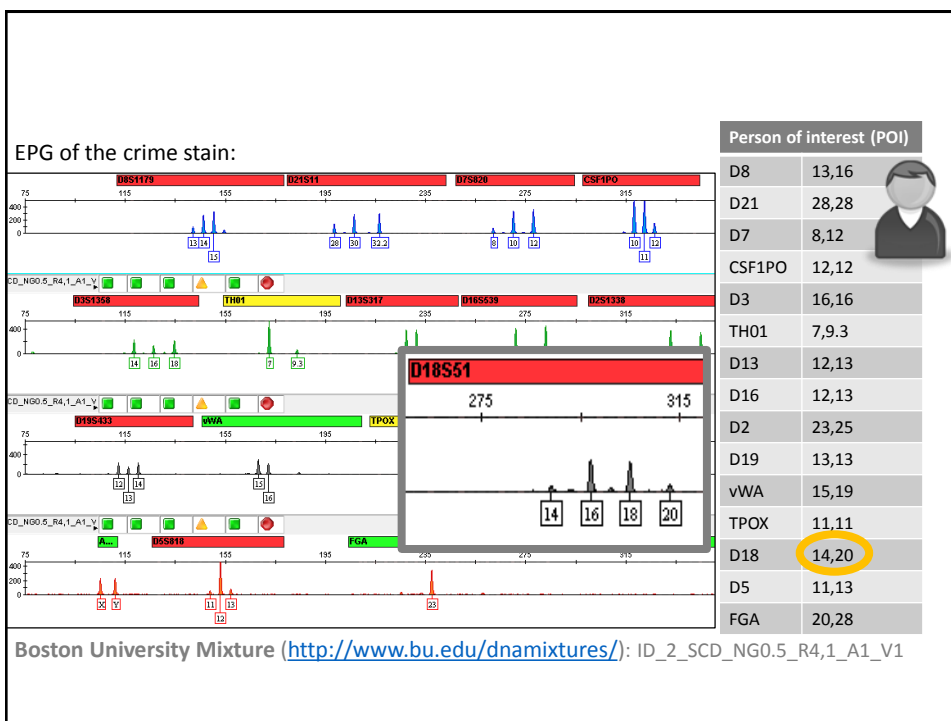
We assume that the major contributor is:

Major Contributor							
D8	D21	D7	CSF	D3	TH01	D13	D16
14,15	30,32,2	10,12	10,11	14,18	7,7	11,12	10,13
D2	D19	vWA	TPOX	D18	D5	FGA	
22,25	12,14	15,16	8,8	16,18	12,12	23,23	

Assumptions

For allelic peaks **above** the stochastic threshold: allele drop-out is impossible (i.e., $D = 0$ and $D_2 = 0$).

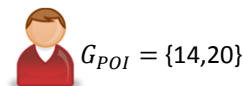
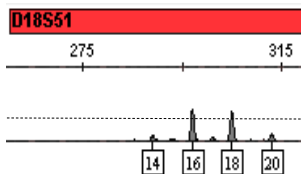
For allelic peaks **below** the stochastic threshold: $D = 0.716$ and $D_2 = 0.256$ (values assigned based on a logistic regression curve and the average allelic peak height of the minor contributor).



Semi-Continuous LR

D18S51

- $p_{14} = 0.134$
- $p_{16} = 0.147$
- $p_{18} = 0.078$
- $p_{20} = 0.018$



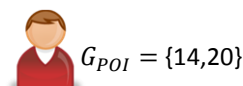
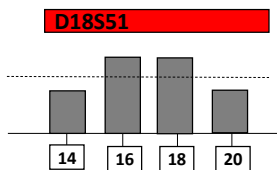
$G_{POI} = \{14,20\}$

stochastic threshold

Semi-Continuous LR

D18S51

- $p_{14} = 0.134$
- $p_{16} = 0.147$
- $p_{18} = 0.078$
- $p_{20} = 0.018$



$G_{POI} = \{14,20\}$

stochastic threshold

Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype $\{14,20\}$?

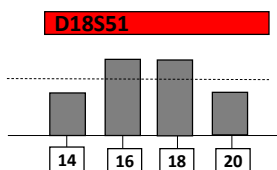
Major	Minor
16,18	14,20

$$\begin{aligned}
 &Pr(16,18) \times Pr(14,20) \\
 &= 2p_{16}p_{18} \times \bar{D} \bar{D} \times 1 \\
 &= \bar{D}^2 \times 2p_{16}p_{18}
 \end{aligned}$$

Semi-Continuous LR

D18S51

$p_{14} = 0.134$
 $p_{16} = 0.147$
 $p_{18} = 0.078$
 $p_{20} = 0.018$



stochastic threshold



$G_{POI} = \{14,20\}$

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

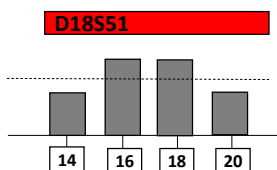
Major	Minor
16,18	14,20

$$\begin{aligned} &Pr(16,18) \times Pr(14,20) \\ &= 2p_{16}p_{18} \times \bar{D} \bar{D} \times 2p_{14}p_{20} \\ &= \bar{D}^2 \times 4p_{14}p_{16}p_{18}p_{20} \end{aligned}$$

Semi-Continuous LR

D18S51

$p_{14} = 0.134$
 $p_{16} = 0.147$
 $p_{18} = 0.078$
 $p_{20} = 0.018$



stochastic threshold



$G_{POI} = \{14,20\}$

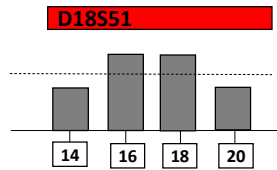
$$\begin{aligned} LR &= \frac{\bar{D}^2 2p_{16}p_{18}}{\bar{D}^2 4p_{14}p_{16}p_{18}p_{20}} \\ &= \frac{1}{2p_{14}p_{20}} \\ &= 207.30 \end{aligned}$$

If we assuming 2 contributors, then allele drop-out is impossible. This makes the $LR_{semi-cont.}$ equal to the LR_{binary} .

Semi-Continuous LR

D18S51

$p_{14} = 0.134$
 $p_{16} = 0.147$
 $p_{18} = 0.078$
 $p_{20} = 0.018$

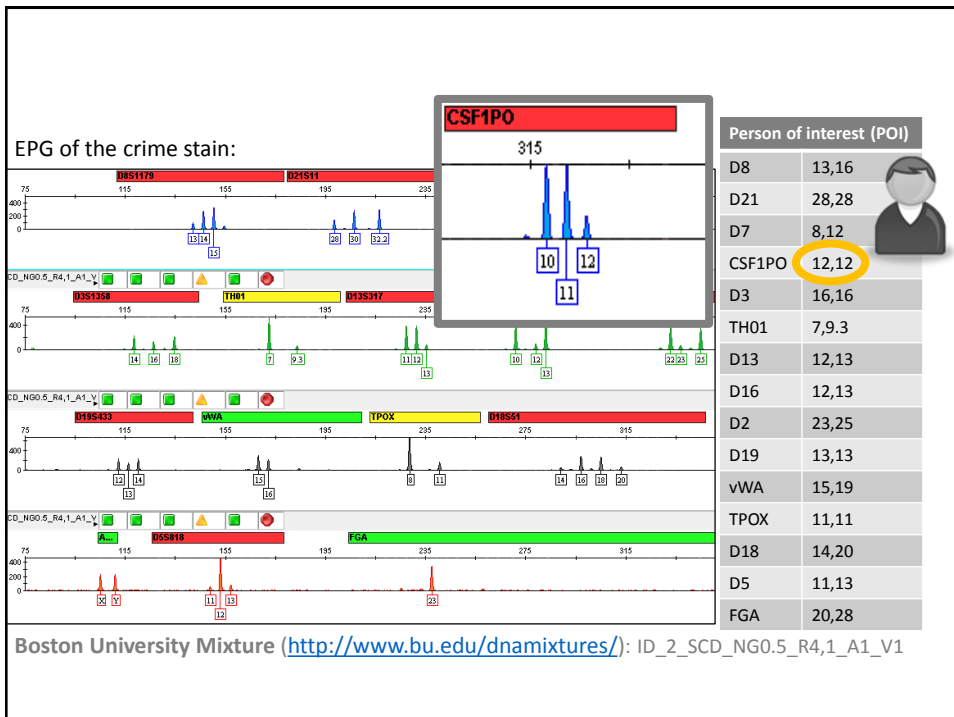


stochastic threshold



$G_{POI} = \{14,20\}$

The DNA typing results are 207 times more probable if the DNA came from the person of interest and an unknown contributor than if the DNA came from two unknown contributors.

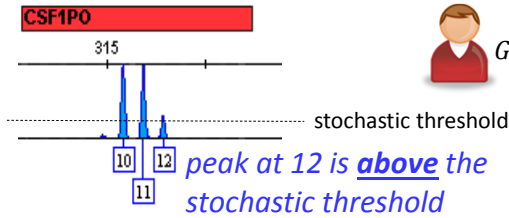


Semi-Continuous LR

If all the peaks are above the stochastic threshold, allele dropout is considered impossible (i.e., $D = 0$ and $D_2 = 0$).

CSF1PO

$p_{10} = 0.220$
 $p_{11} = 0.309$
 $p_{12} = 0.360$

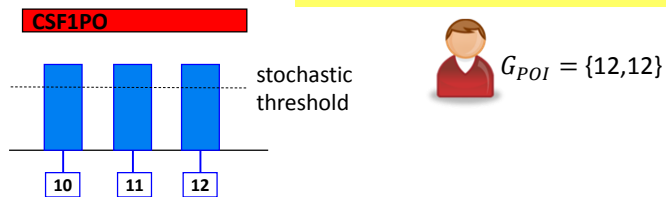


Semi-Continuous LR

If all the peaks are above the stochastic threshold, allele dropout is considered impossible (i.e., $D = 0$ and $D_2 = 0$).

CSF1PO

$p_{10} = 0.220$
 $p_{11} = 0.309$
 $p_{12} = 0.360$



Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype {12,12}?

Major	Minor
10,11	12,12
10,11	10,12
10,11	11,12

$$\begin{aligned}
 &Pr(10,11) \times Pr(12,12) \\
 &= 2p_{10}p_{11} \times 1 \\
 &= 2p_{10}p_{11}
 \end{aligned}$$

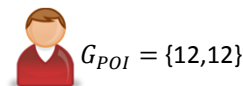
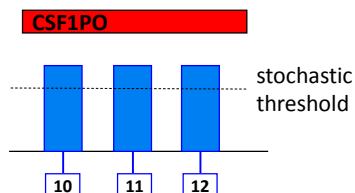
Semi-Continuous LR

CSF1PO

$$p_{10} = 0.220$$

$$p_{11} = 0.309$$

$$p_{12} = 0.360$$



Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

Major	Minor
10,11	12,12
10,11	10,12
10,11	11,12

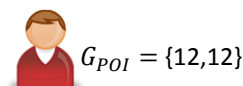
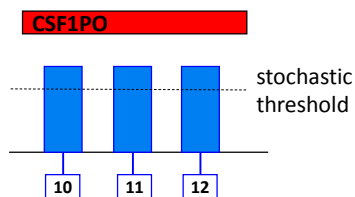
Semi-Continuous LR

CSF1PO

$$p_{10} = 0.220$$

$$p_{11} = 0.309$$

$$p_{12} = 0.360$$



Denominator:

Major	Minor
10,11	12,12
10,11	10,12
10,11	11,12

$$Pr(10,11) \times Pr(12,12) + Pr(10,11) \times Pr(10,12) + Pr(10,11) \times Pr(11,12)$$

$$= 2p_{10}p_{11} \times p_{12}^2 + 2p_{10}p_{11} \times 2p_{10}p_{12}$$

$$+ 2p_{10}p_{11} \times 2p_{11}p_{12}$$

$$= 2p_{10}p_{11}p_{12}(p_{12} + 2p_{10} + 2p_{11})$$

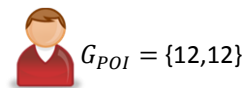
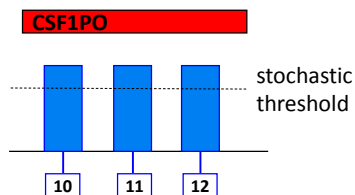
Semi-Continuous LR

CSF1PO

$$p_{10} = 0.220$$

$$p_{11} = 0.309$$

$$p_{12} = 0.360$$



$$LR = \frac{2p_{10}p_{11}}{2p_{10}p_{11}p_{12}(p_{12} + 2p_{10} + 2p_{11})}$$

$$= \frac{1}{p_{12}(p_{12} + 2p_{10} + 2p_{11})}$$

$$= 1.96$$

Since allele drop-out is impossible, the $LR_{semi-cont.}$ is equal to the LR_{binary} .

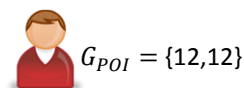
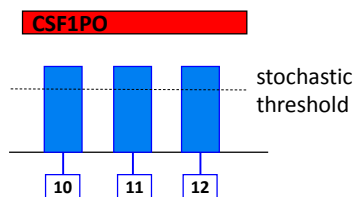
Semi-Continuous LR

CSF1PO

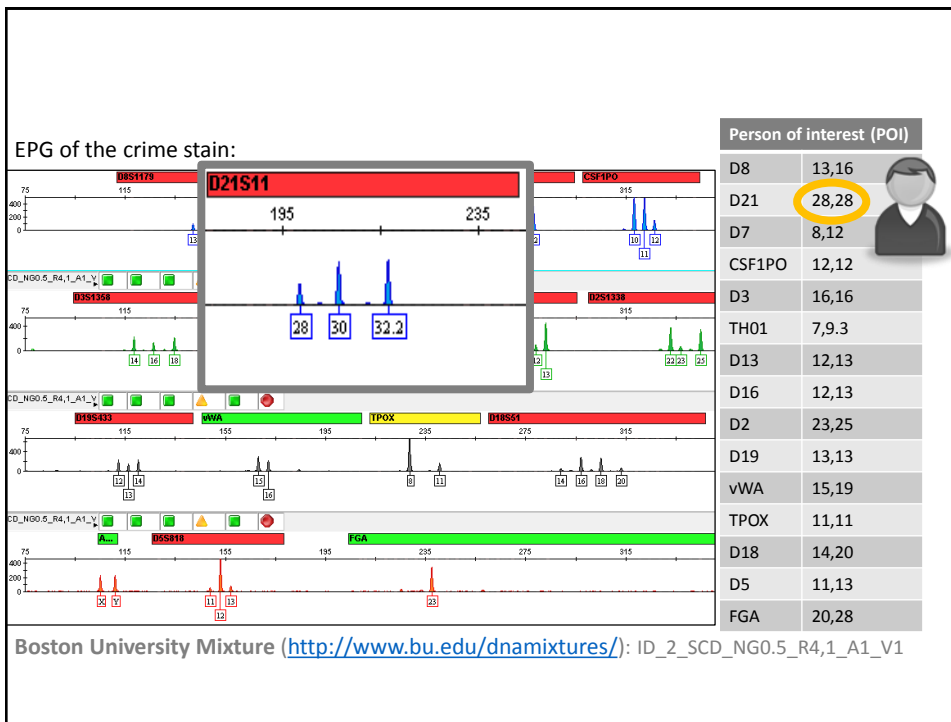
$$p_{10} = 0.220$$

$$p_{11} = 0.309$$

$$p_{12} = 0.360$$



The DNA typing results are about 2 times more probable if the DNA came from the person of interest and an unknown contributor than if the DNA came from two unknown contributors.



Semi-Continuous LR

D21S11

$$p_{28} = 0.159$$

$$p_{30} = 0.283$$

$$p_{32.2} = 0.090$$



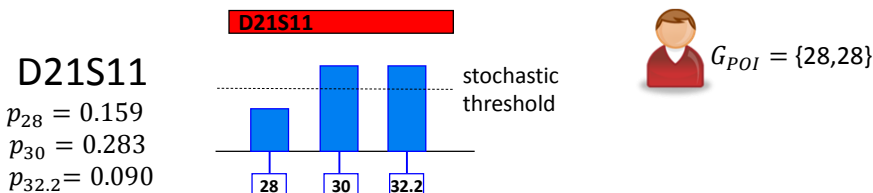
peak at 28 is below the stochastic threshold



$$G_{POI} = \{28,28\}$$

stochastic threshold

Semi-Continuous LR



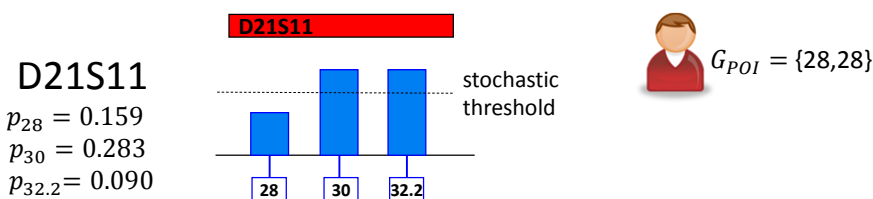
Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is** a contributor and the POI has genotype {28,28}?

Major	Minor
30,32.2	28,F

$$\begin{aligned}
 &Pr(30,32.2) \times Pr(28,28) \\
 &= 2p_{30}p_{32.2} \times \overline{D}_2 \times 1 \\
 &= \overline{D}_2 \times 2p_{30}p_{32.2}
 \end{aligned}$$

Semi-Continuous LR



Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

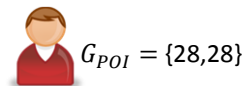
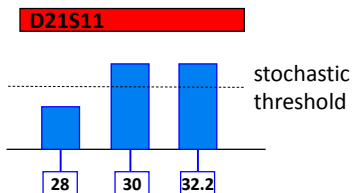
Major	Minor
30,32.2	28,F

$$\begin{aligned}
 &Pr(30,32.2) \times Pr(28, F) \\
 &= 2p_{30}p_{32.2} \times [\overline{D}D \times 2p_{28}p_Q \\
 &\quad + \overline{D} \times 2p_{28}(p_{30} + p_{32.2}) + \overline{D}_2 \times p_{28}^2]
 \end{aligned}$$

Semi-Continuous LR

D21S11

$p_{28} = 0.159$
 $p_{30} = 0.283$
 $p_{32.2} = 0.090$



$$LR = \frac{\overline{D}_2 \times 2p_{30}p_{32.2}}{2p_{30}p_{32.2} \times [\overline{D}D \times 2p_{28}p_Q + \overline{D} \times 2p_{28}(p_{30} + p_{32.2}) + \overline{D}_2 \times p_{28}^2]}$$

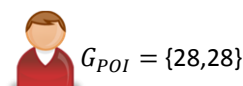
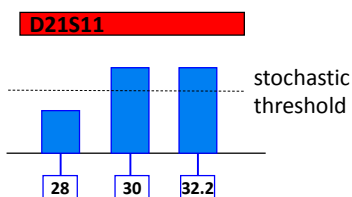
$$= \frac{\overline{D}_2}{\overline{D}D \times 2p_{28}p_Q + \overline{D} \times 2p_{28}(p_{30} + p_{32.2}) + \overline{D}_2 \times p_{28}^2}$$

$$= 8.99$$

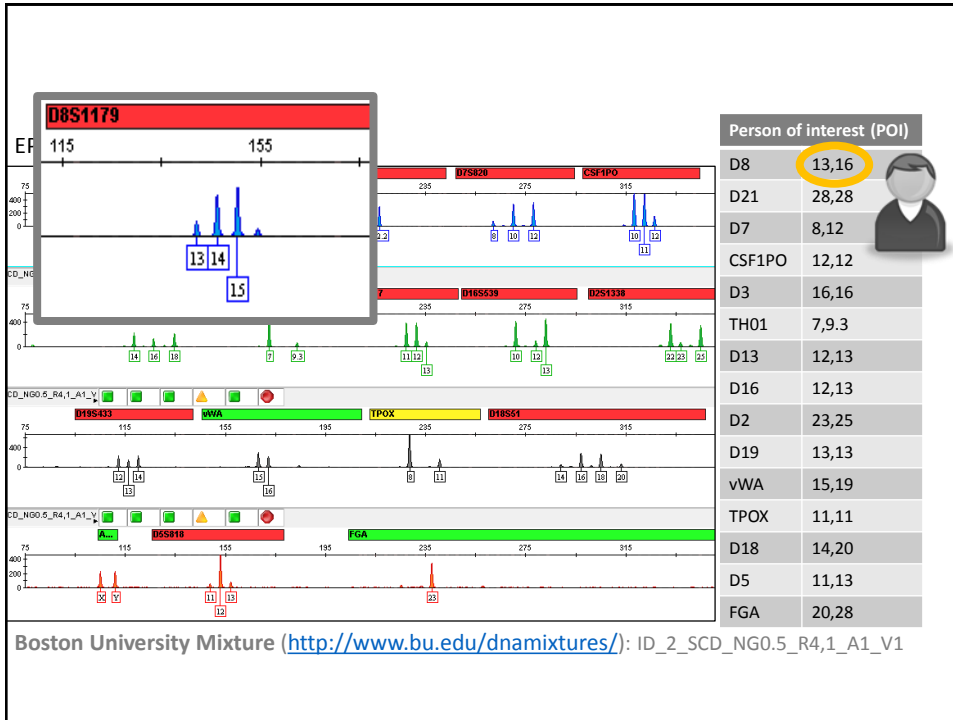
Semi-Continuous LR

D21S11

$p_{28} = 0.159$
 $p_{30} = 0.283$
 $p_{32.2} = 0.090$



The DNA typing results are about 9 times more probable if the DNA came from the person of interest and an unknown contributor than if the DNA came from two unknown contributors.



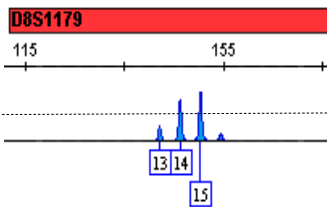
Semi-Continuous LR

D8S1179

$$p_{13} = 0.330$$

$$p_{14} = 0.166$$

$$p_{15} = 0.104$$



$G_{POI} = \{13,16\}$

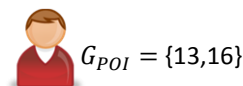
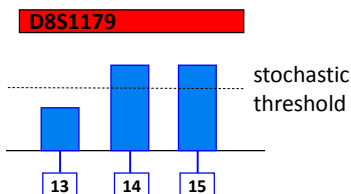
stochastic threshold

peak at 13 is below the stochastic threshold

Semi-Continuous LR

D8S1179

$p_{13} = 0.330$
 $p_{14} = 0.166$
 $p_{15} = 0.104$



Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is** a contributor and the POI has genotype {13,16}?

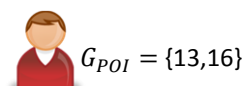
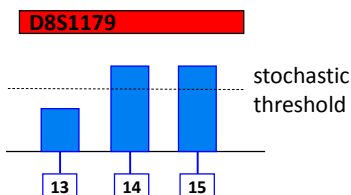
Major	Minor
14,15	13,F

$$\begin{aligned} &Pr(14,15) \times Pr(13,16) \\ &= 2p_{14}p_{15} \times \bar{D}D \times 1 \\ &= \bar{D}D \times 2p_{14}p_{15} \end{aligned}$$

Semi-Continuous LR

D8S1179

$p_{13} = 0.330$
 $p_{14} = 0.166$
 $p_{15} = 0.104$



Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

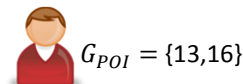
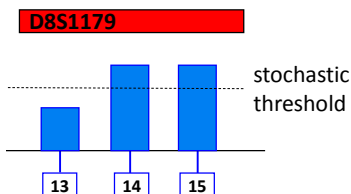
Major	Minor
14,15	13,F

$$\begin{aligned} &Pr(14,15) \times Pr(13, F) \\ &= 2p_{14}p_{15} \times [\bar{D}D \times 2p_{13}p_Q \\ &\quad + \bar{D} \times 2p_{13}(p_{14} + p_{15}) + \bar{D}_2 \times p_{13}^2] \end{aligned}$$

Semi-Continuous LR

D8S1179

$p_{13} = 0.330$
 $p_{14} = 0.166$
 $p_{15} = 0.104$



$$LR = \frac{\bar{D}D \times 2p_{14}p_{15}}{2p_{14}p_{15} \times [\bar{D}D \times 2p_{13}p_Q + \bar{D} \times 2p_{13}(p_{14} + p_{15}) + \bar{D}_2 \times p_{13}^2]}$$

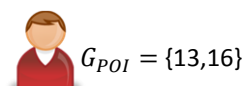
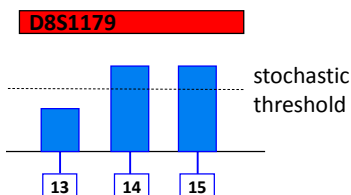
$$= \frac{\bar{D}D}{\bar{D}D \times 2p_{13}p_Q + \bar{D} \times 2p_{13}(p_{14} + p_{15}) + \bar{D}_2 \times p_{13}^2}$$

$$= 1.10$$

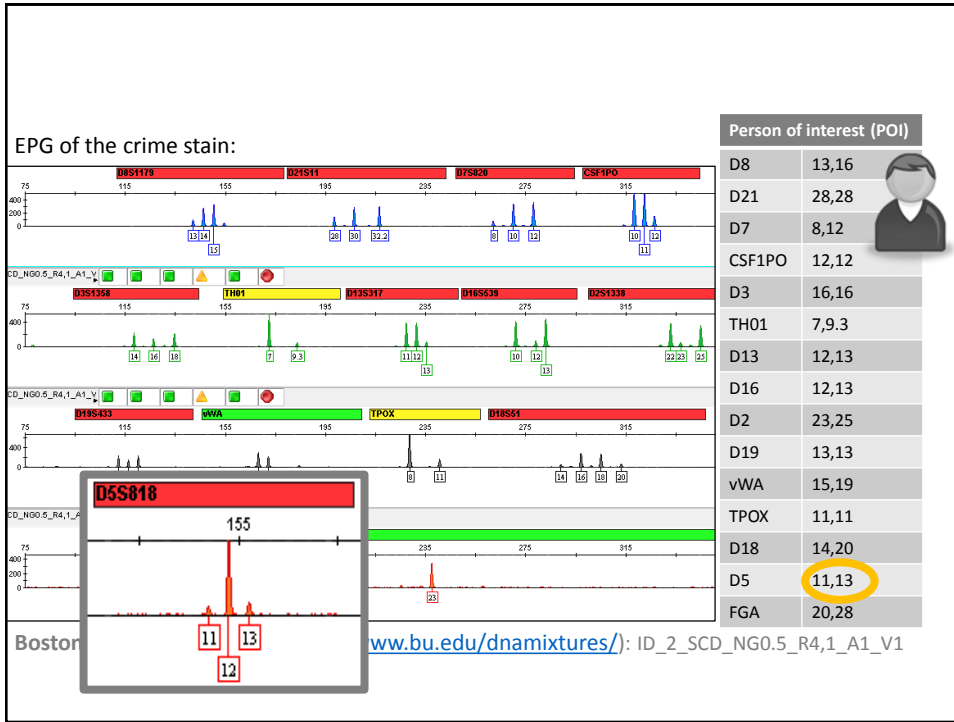
Semi-Continuous LR

D8S1179

$p_{13} = 0.330$
 $p_{14} = 0.166$
 $p_{15} = 0.104$



The DNA typing results are about equally probable if the DNA came from the person of interest and an unknown contributor than if the DNA came from two unknown contributors.



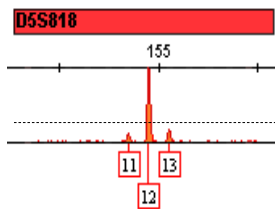
Semi-Continuous LR

D5S818

$$p_{11} = 0.356$$

$$p_{12} = 0.388$$

$$p_{13} = 0.143$$

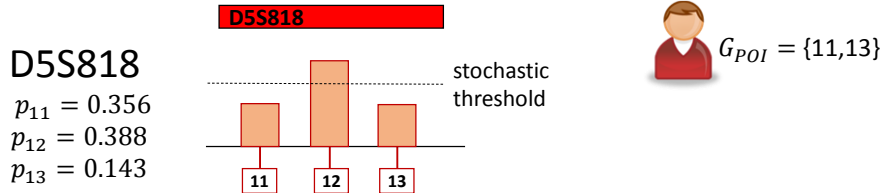


$$G_{POI} = \{11,13\}$$

stochastic
threshold

peaks at 11 and 13 are below
the stochastic threshold

Semi-Continuous LR



Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype {11,13}?

Major	Minor
12,12	11,13

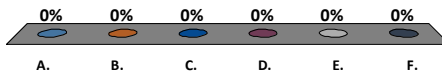
$$Pr(12,12) \times Pr(11,13)$$

$$= \dots$$

What is the numerator of the likelihood ratio?

- A. $\bar{D} \times p_{12}^2$
- B. $\bar{D}_2 \times p_{12}^2$
- C. $\bar{D}^2 \times p_{12}^2$
- D. $\bar{D}D \times p_{12}^2$
- E. I'm lost.

Response Counter



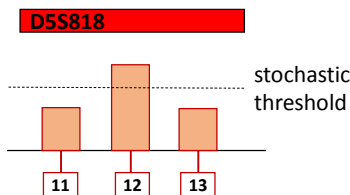
Semi-Continuous LR

D5S818

$$p_{11} = 0.356$$

$$p_{12} = 0.388$$

$$p_{13} = 0.143$$



$G_{POI} = \{11,13\}$

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

Major	Minor
12,12	11,13

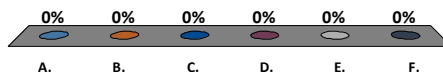
$$Pr(12,12) \times Pr(11,13)$$

= ...

What is the denominator of the likelihood ratio?

- A. $p_{12}^2 \times \bar{D} \times 2p_{11}p_{13}$
- B. $p_{12}^2 \times \bar{D}_2 \times 2p_{11}p_{13}$
- C. $p_{12}^2 \times \bar{D}^2 \times 2p_{11}p_{13}$
- D. $p_{12}^2 \times \bar{D}D \times 2p_{11}p_{13}$
- E. I'm lost.

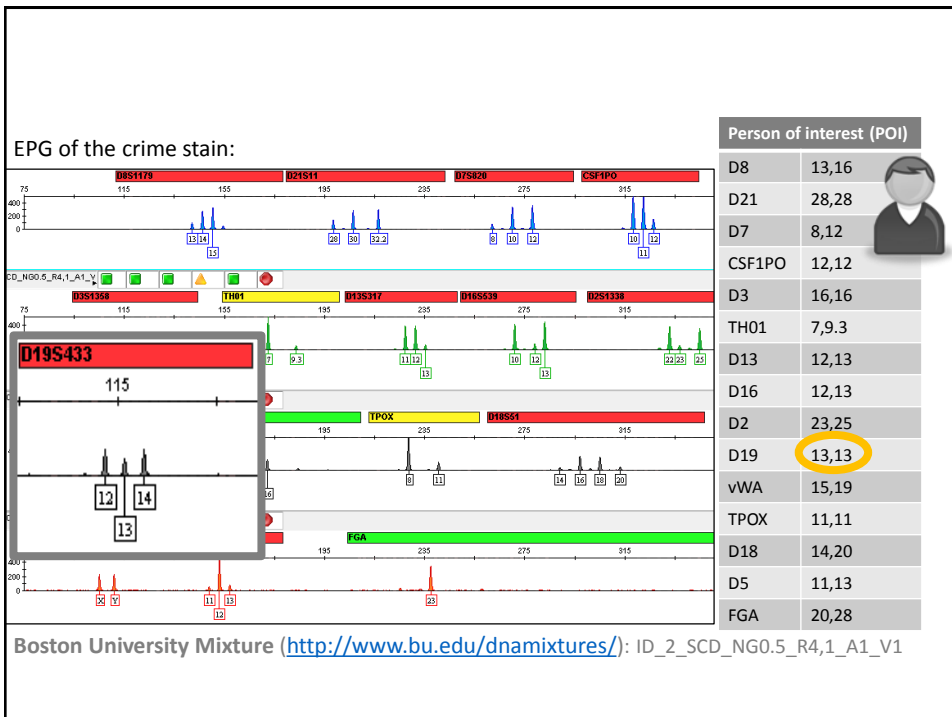
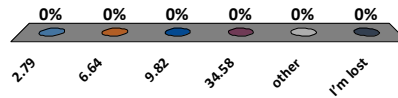
Response Counter



What is the likelihood ratio?

- A. 2.79
- B. 6.64
- C. 9.82
- D. 34.58
- E. I'm lost

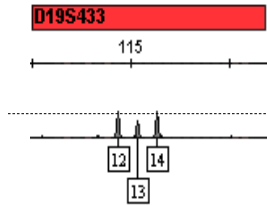
Response Counter



Semi-Continuous LR

D19S433

$p_{12} = 0.071$
 $p_{13} = 0.255$
 $p_{14} = 0.361$



$G_{POI} = \{13,13\}$

stochastic threshold

peak at 13 is **below** the stochastic threshold

Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is** a contributor and the POI has genotype {13,13}?

Major	Minor
12,14	13,F

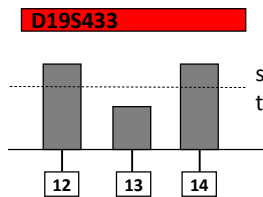
$$Pr(12,14) \times Pr(13,13)$$

$$= \dots$$

Semi-Continuous LR

D19S433

$p_{12} = 0.071$
 $p_{13} = 0.255$
 $p_{14} = 0.361$



$G_{POI} = \{13,13\}$

stochastic threshold

Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is** a contributor and the POI has genotype {13,13}?

Major	Minor
12,14	13,F

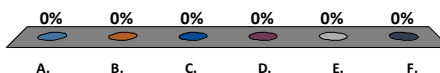
$$Pr(12,14) \times Pr(13,13)$$

$$= \dots$$

What is the numerator of the likelihood ratio?

- A. $\bar{D} \times 2p_{12}p_{14}$
- B. $\bar{D}_2 \times 2p_{12}p_{14}$
- C. $\bar{D}^2 \times 2p_{12}p_{14}$
- D. $\bar{D}D \times 2p_{12}p_{14}$
- E. I'm lost.

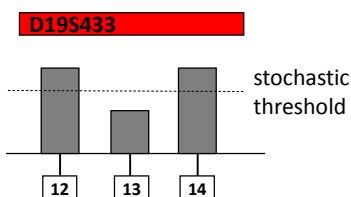
Response Counter




Semi-Continuous LR

D19S433

$p_{12} = 0.071$
 $p_{13} = 0.255$
 $p_{14} = 0.361$



 $G_{POI} = \{13,13\}$

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

Major	Minor
12,14	13,F

$$Pr(12,14) \times Pr(13,F)$$

$$= \dots$$

What is the denominator of the likelihood ratio?

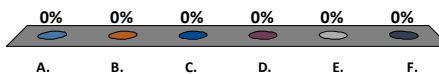
- A. $2p_{12}p_{14} \times \bar{D}_2p_{13}^2$
- B. $2p_{12}p_{14} \times p_{13}(p_{13} + 2p_{12} + 2p_{14})$
- C. $2p_{12}p_{14} \times [\bar{D}_2p_{13}^2 + \bar{D}2p_{13}(p_{12} + p_{14}) + \bar{D}D2p_{13}p_Q]$
- D. $2p_{12}p_{14} \times [\bar{D}_2p_{13}^2 + \bar{D}D2p_{13}p_Q]$
- E. *I'm lost.*

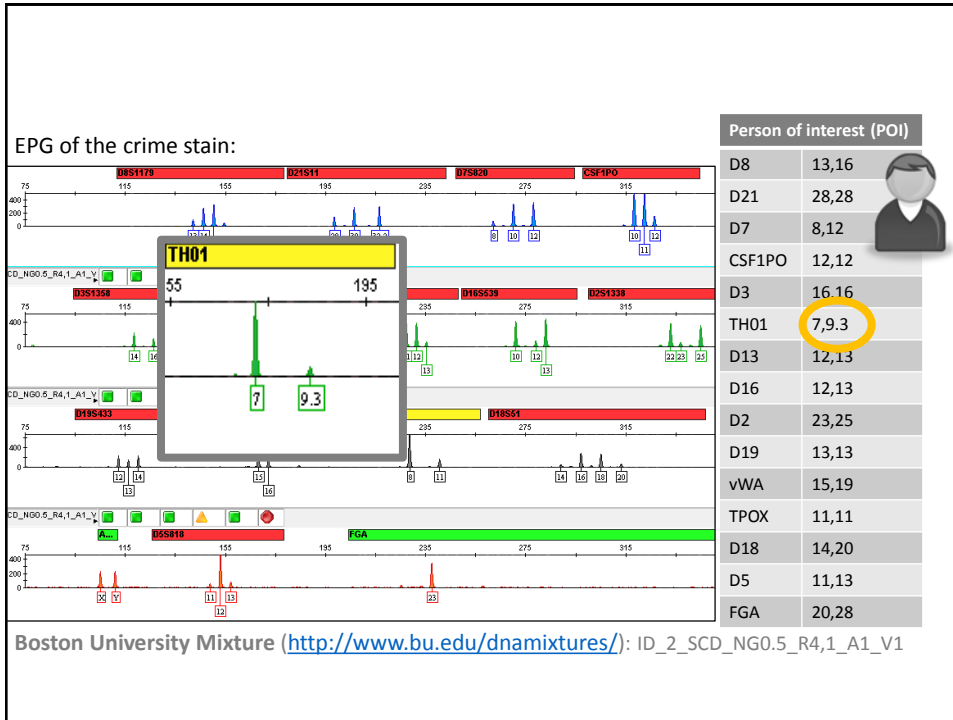
Response
Counter

What is the likelihood ratio?

- A. 3.50
- B. 5.19
- C. 9.20
- D. 15.38
- E. *I'm lost.*

Response
Counter



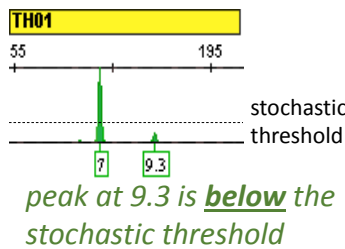


Semi-Continuous LR

TH01

$$p_7 = 0.194$$

$$p_{9.3} = 0.345$$



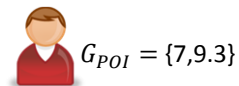
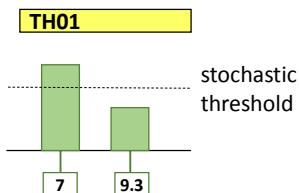
$$G_{POI} = \{7,9.3\}$$

Semi-Continuous LR

TH01

$$p_7 = 0.194$$

$$p_{9.3} = 0.345$$



$G_{POI} = \{7,9.3\}$

Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype $\{7,9.3\}$?

Major	Minor
7,7	9.3,F

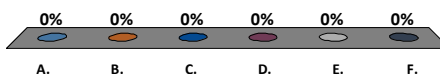
$$Pr(7,7) \times Pr(7,9.3)$$

$$= \dots$$

What is the numerator of the likelihood ratio?

- A. $\bar{D} \times p_7^2$
- B. $\bar{D}_2 \times p_7^2$
- C. $\bar{D}^2 \times p_7^2$
- D. $\bar{D}D \times p_7^2$
- E. I'm lost.

Response Counter

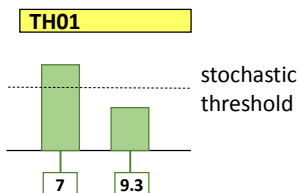


Semi-Continuous LR

TH01

$$p_7 = 0.194$$

$$p_{9.3} = 0.345$$



$$G_{POI} = \{7,9.3\}$$

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

Major	Minor
7,7	9.3,F

$$Pr(7,7) \times Pr(9.3, F)$$

$$= \dots$$

What is the denominator of the likelihood ratio?

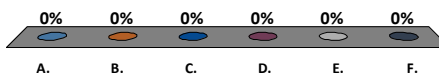
- A. $p_7^2 \times [\overline{D}_2 p_{9.3}^2 + \overline{D} 2 p_{9.3} p_7 + \overline{D} D 2 p_{9.3} p_Q]$
- B. $p_7^2 \times p_{9.3} (p_{9.3} + 2 p_7)$
- C. $p_7^2 \times [\overline{D}_2 p_{9.3}^2 + \overline{D} 2 p_{9.3} (p_7 + p_Q)]$
- D. $p_7^2 \times \overline{D}_2 p_{9.3}^2$
- E. I'm lost.

Response Counter

What is the likelihood ratio?

- A. 1.48
- B. 3.95
- C. 11.29
- D. 26.30
- E. I'm lost.

Response
Counter



Semi-Continuous LR


probability of allele drop-in

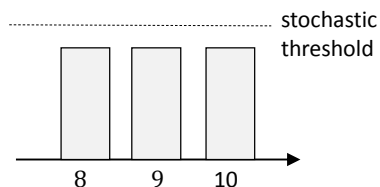
$$Pr(drop - in) = C$$

$$Pr(not drop - in) = 1 - C = \bar{C}$$

based on the notation in:
J. Buckleton, C.M. Triggs, S.J. Walsh. (2005). *Forensic DNA Evidence Interpretation*. CRC Press, London.

Semi-Continuous LR

Examples: 
 {8,9}




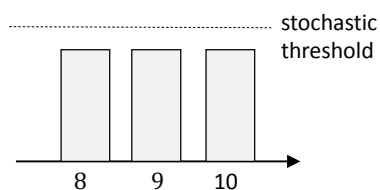
Assumptions: 1 contributor, maximum 1 drop-in allele per locus, and no tri-allelic contributor

Numerator:

$$Pr(G_{CS}|G_K, H_p) = \bar{D} \bar{D} C p_{10} \times 1 = \bar{D}^2 C p_{10}$$

Semi-Continuous LR

Examples: 
 {8,9}




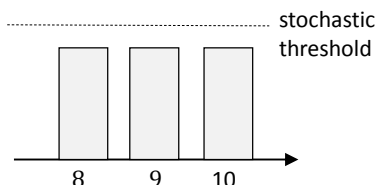
Assumptions: 1 contributor, maximum 1 drop-in allele per locus, and no tri-allelic contributor

Denominator:

$$\begin{aligned} Pr(G_{CS}|G_K, H_d) &= \bar{D} \bar{D} C p_{10} \times 2p_8p_9 + \bar{D} \bar{D} C p_9 \times 2p_8p_{10} \\ &\quad + \bar{D} \bar{D} C p_8 \times 2p_9p_{10} \\ &= \bar{D}^2 C \times 6p_8p_9p_{10} \end{aligned}$$

Semi-Continuous LR

Examples: 
 {8,9}



Assumptions: 1 contributor, maximum 1 drop-in allele per locus, and no tri-allelic contributor

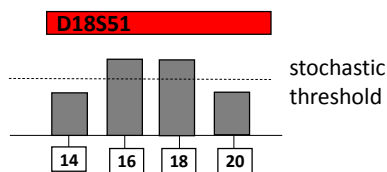
$$LR = \frac{\bar{D}^2 C p_{10}}{\bar{D}^2 C \times 6p_8 p_9 p_{10}}$$


$$= \frac{1}{6p_8 p_9}$$

Semi-Continuous LR with possibility of allele drop-in

D18S51

$p_{14} = 0.134$
 $p_{16} = 0.147$
 $p_{18} = 0.078$
 $p_{20} = 0.018$



 $G_{POI} = \{14,20\}$

Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype {14,20}?

Major	Minor
16,18	14,20

$$Pr(16,18) \times Pr(14,20)$$

$$= 2p_{16}p_{18} \times \bar{D} \bar{D} \bar{C} \times 1$$

$$= \bar{D}^2 \bar{C} \times 2p_{16}p_{18}$$

Semi-Continuous LR with possibility of allele drop-in

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI **is not** a contributor?

Major	Minor	
16,18	14,20	$Pr(16,18) \times [Pr(14,20) + Pr(14,14) + Pr(14,16) + Pr(14,18) + Pr(14, Q) + Pr(20,20) + Pr(16,20) + Pr(18,20) + Pr(20, Q)]$ $= 2p_{16}p_{18} \times [\bar{D} \bar{D} \bar{C} \times 2p_{14}p_{20} + \bar{D}_2 C p_{20} \times p_{14}^2 + \bar{D} C p_{20} \times 2p_{14}p_{16} + \bar{D} C p_{20} \times 2p_{14}p_{18} + \bar{D} D C p_{20} \times 2p_{14}p_Q + \bar{D}_2 C p_{14} \times p_{20}^2 + \bar{D} C p_{14} \times 2p_{16}p_{20} + \bar{D} C p_{14} \times 2p_{18}p_{20} + \bar{D} D C p_{14} \times 2p_{20}p_Q]$
16,18	14,14	
16,18	14,16	
16,18	14,18	
16,18	14,Q	
16,18	20,20	
16,18	16,20	
16,18	18,20	
16,18	20,Q	

Semi-Continuous LR with possibility of allele drop-in

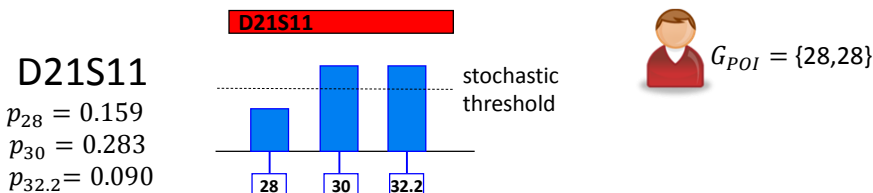
$$LR = \frac{\bar{D}^2 \bar{C} \times 2p_{16}p_{18}}{2p_{16}p_{18} \times [\bar{D}^2 \bar{C} \times 2p_{14}p_{20} + \bar{D}_2 C p_{20} \times p_{14}^2 + \bar{D} C p_{20} \times 2p_{14}(p_{16} + p_{18}) + \bar{D} D C p_{20} \times 2p_{14}p_Q + \bar{D}_2 C p_{14} \times p_{20}^2 + \bar{D} C p_{14} \times 2p_{20}(p_{16} + p_{18}) + \bar{D} D C p_{14} \times 2p_{20}p_Q]}$$

$$= \frac{\bar{D}^2 \bar{C}}{\bar{D}^2 \bar{C} \times 2p_{14}p_{20} + \bar{D}_2 C p_{20} \times p_{14}^2 + \bar{D} C p_{20} \times 2p_{14}(p_{16} + p_{18}) + \bar{D} D C p_{20} \times 2p_{14}p_Q + \bar{D}_2 C p_{14} \times p_{20}^2 + \bar{D} C p_{14} \times 2p_{20}(p_{16} + p_{18}) + \bar{D} D C p_{14} \times 2p_{20}p_Q}$$

$C = 0.01: \quad LR = 196.52$

$C = 0.05: \quad LR = 161.24$

Semi-Continuous LR with possibility of allele drop-in



Numerator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is a contributor and the POI has genotype {28,28}?

Major	Minor
30,32.2	28,F

$$\begin{aligned}
 &Pr(30,32.2) \times Pr(28,28) \\
 &= 2p_{30}p_{32.2} \times \overline{D_2}\overline{C} \times 1 \\
 &= \overline{D_2}\overline{C} \times 2p_{30}p_{32.2}
 \end{aligned}$$

Semi-Continuous LR with possibility of allele drop-in

Denominator:

What is the probability of obtaining these DNA typing results for the crime stain if the POI is not a contributor?

Major	Minor
30,32.2	28,28
30,32.2	28,30
30,32.2	28,32.2
30,32.2	28,Q
30,32.2	30,30
30,32.2	30,32.2
30,32.2	32.2,32.2
30,32.2	30,Q
30,32.2	32.2,Q
30,32.2	Q,Q or Q,Q'

$$\begin{aligned}
 &Pr(30,32.2) \times [Pr(28,28) + Pr(28,30) \\
 &+ Pr(28,32.2) + Pr(28,Q) + Pr(30,30) \\
 &+ Pr(30,32.2) + Pr(32.2,32.2) + Pr(30,Q) \\
 &+ Pr(32.2,Q) + Pr(Q,Q)] \\
 &= 2p_{30}p_{32.2} \times [\overline{D_2}\overline{C} \times p_{28}^2 + \overline{D_2}\overline{C} \times 2p_{28}(p_{30} + p_{32.2}) \\
 &+ \overline{D_2}D\overline{C} \times 2p_{28}p_Q + Cp_{28} \times (p_{14} + p_{15})^2 + DCp_{28} \\
 &\times 2(p_{30} + p_{32.2})p_Q + D_2Cp_{28} \times p_Q^2 + D^2Cp_{28}
 \end{aligned}$$

Semi-Continuous LR with possibility of allele drop-in

$$\begin{aligned}
 LR &= \frac{\overline{D_2\bar{C}} \times 2p_{30}p_{32.2}}{2p_{30}p_{32.2} \times [\overline{D_2\bar{C}} \times p_{28}^2 + \overline{D\bar{C}} \times 2p_{28}(p_{30} + p_{32.2}) + \overline{DD\bar{C}} \times 2p_{28}p_Q \\
 &+ Cp_{28} \times (p_{14} + p_{15})^2 + DCp_{28} \times 2(p_{30} + p_{32.2})p_Q + D_2Cp_{28} \times p_Q^2 + D^2Cp_{28} \\
 &\times 2p_Qp_{2Q}] \\
 &= \frac{\overline{D_2\bar{C}}}{\overline{D_2\bar{C}} \times p_{28}^2 + \overline{D\bar{C}} \times 2p_{28}(p_{30} + p_{32.2}) + \overline{DD\bar{C}} \times 2p_{28}p_Q + Cp_{28} \\
 &\times (p_{14} + p_{15})^2 + DCp_{28} \times 2(p_{30} + p_{32.2})p_Q + D_2Cp_{28} \times p_Q^2 + D^2Cp_{28} \times 2p_Qp_{2Q}}
 \end{aligned}$$