

Mixture Interpretation & Statistics Workshop

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Disclaimer

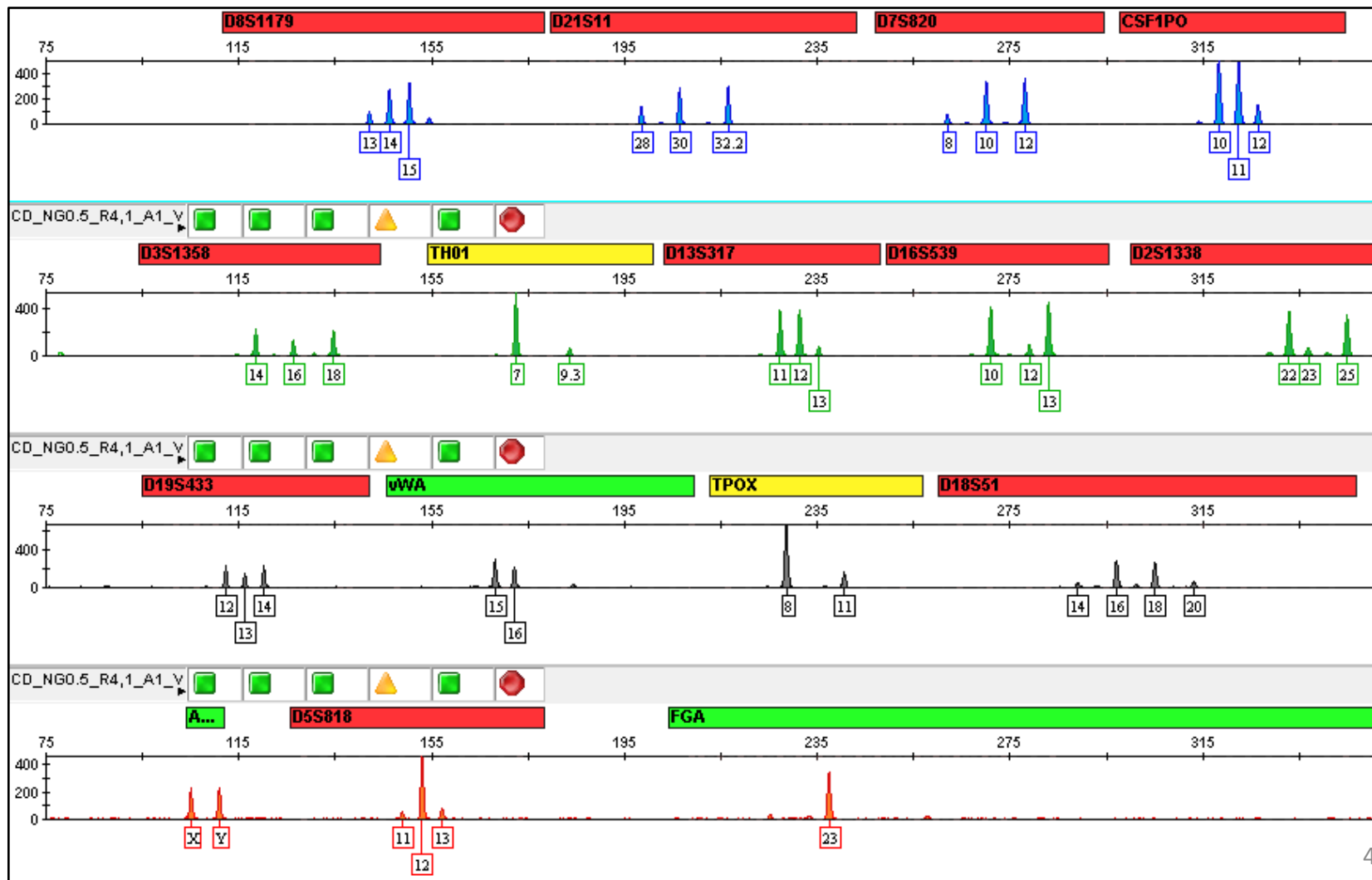
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Overview

Likelihood Ratio

- Binary Approach
- Semi-Continuous Approach
- Continuous Approach



stochastic threshold = 150 rfu

analytical threshold = 30 rfu

CPI

DANGER ZONE

DANGER ZONE

stochastic threshold = 150 rfu

analytical threshold = 30 rfu

CPI

DANGER ZONE

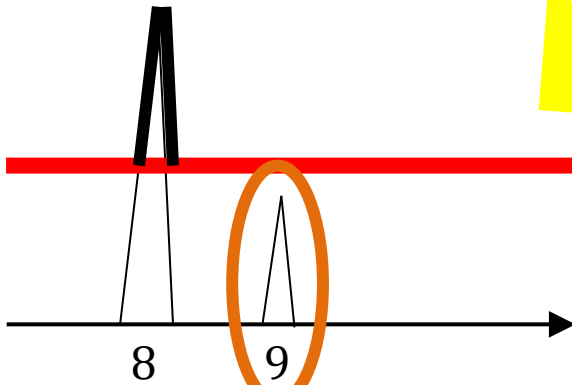
DANGER ZONE

Stochastic Effects



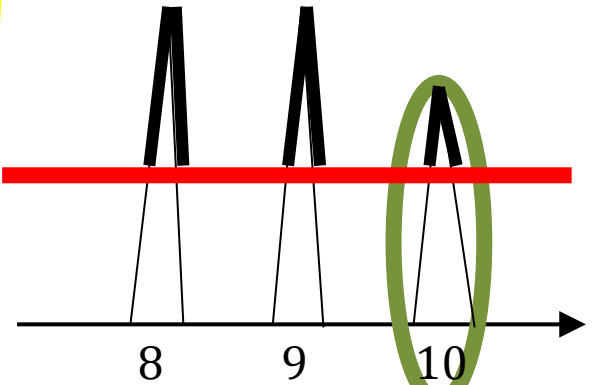
{8,9}

allele drop-out



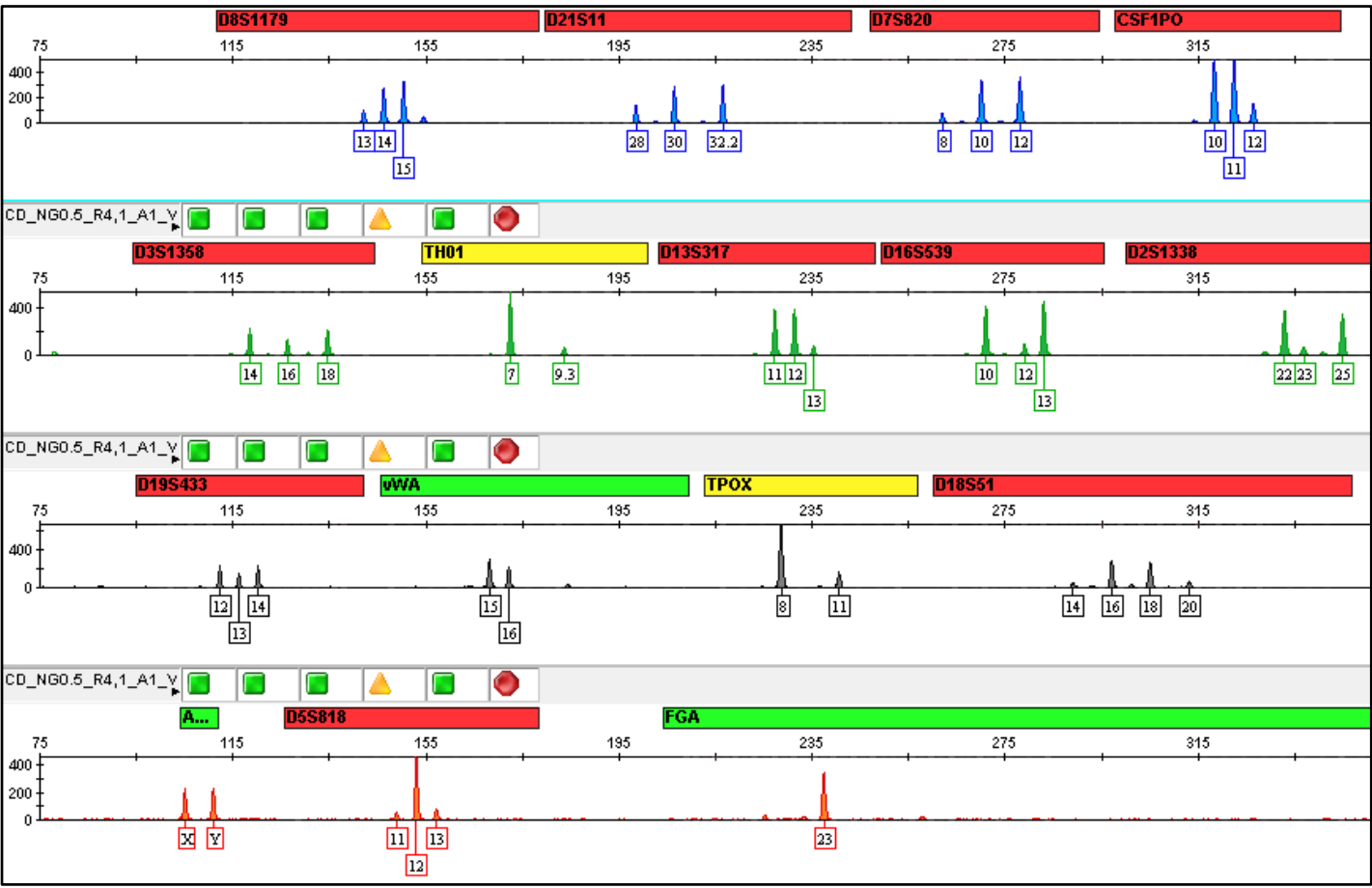
drop-out of allele 9

allele drop-in

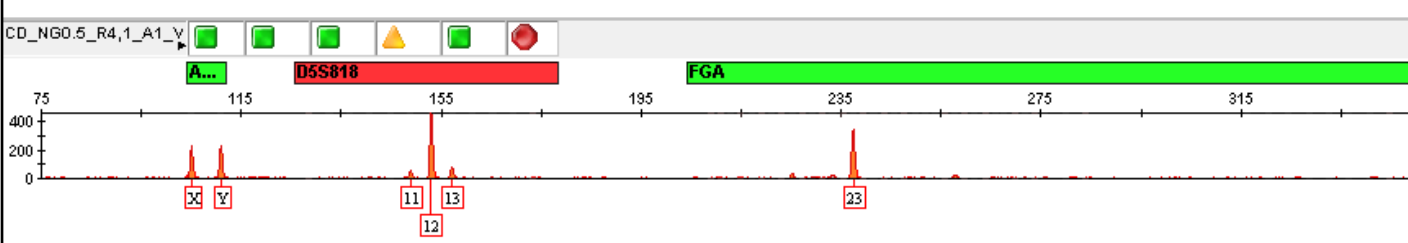
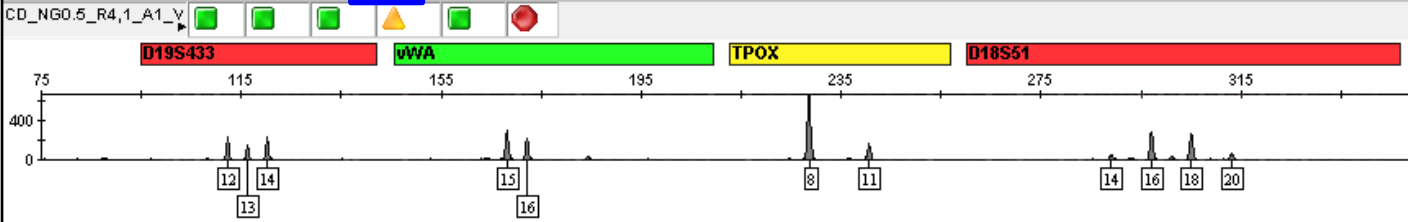
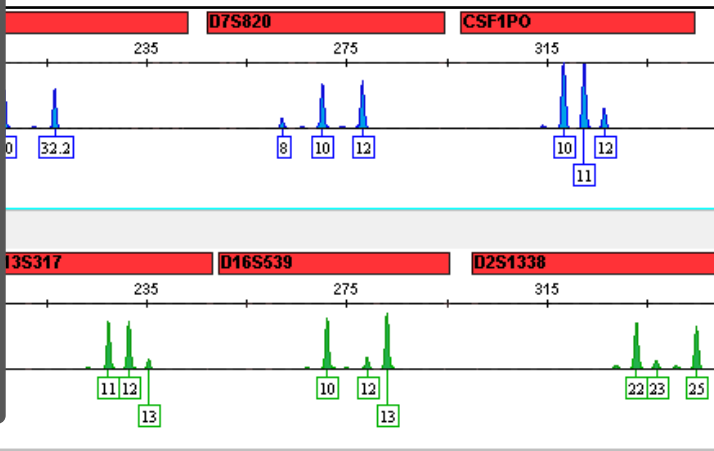
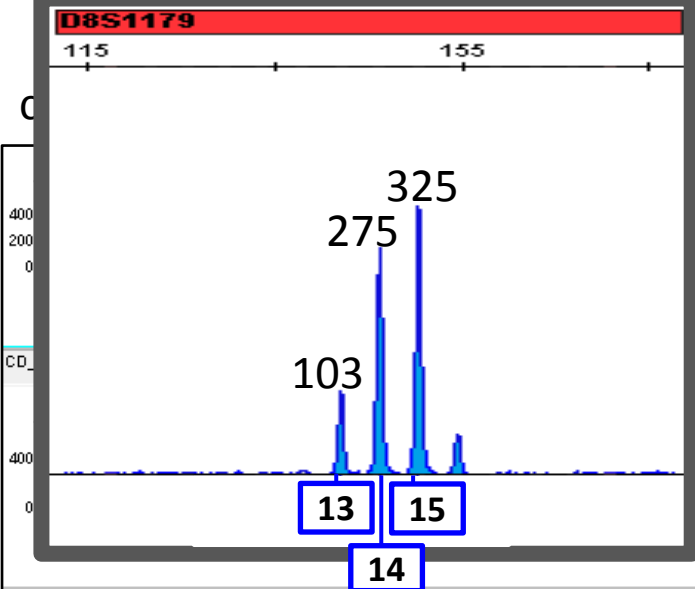


drop-in of allele 10

crime stain:



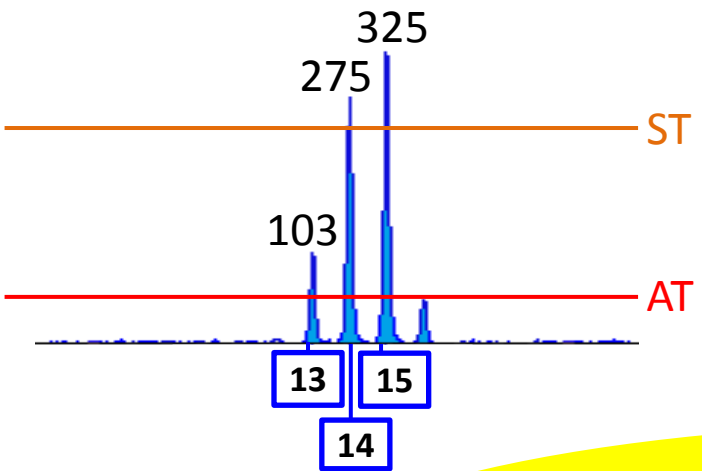
Person of interest (POI)		
D8	13,16	
D21	28,28	
D7	8,12	
CSF1PO	12,12	
D3	16,16	
TH01	7,9.3	
D13	12,13	
D16	12,13	
D2	23,25	
D19	13,13	
vWA	15,19	
TPOX	11,11	
D18	14,20	
D5	11,13	
FGA	20,28	8



Person of interest (POI)		
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D13	12,13	
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D18	14,20	
D5	11,13	
FGA	20,28	9

D8S1179

115 155



person of interest (POI)

{13,16}

Is the person of interest (POI) a contributor to this mixture?

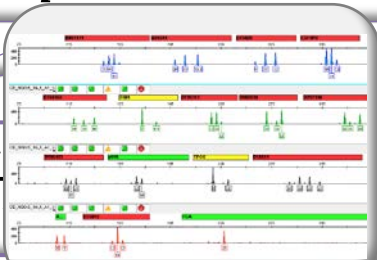


There are two sides to every story...



prosecution's
proposition

H_p : POI is a contributor.



H_d : POI is not a contributor.



forensic
scientist

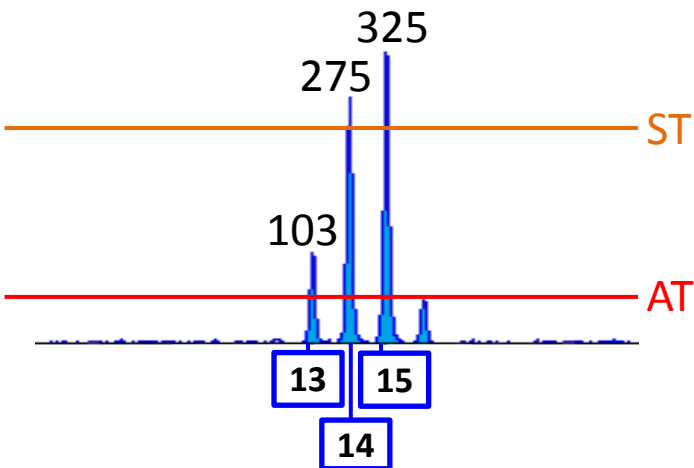


defense's
proposition

D8S1179

115 155

Role of the forensic scientist



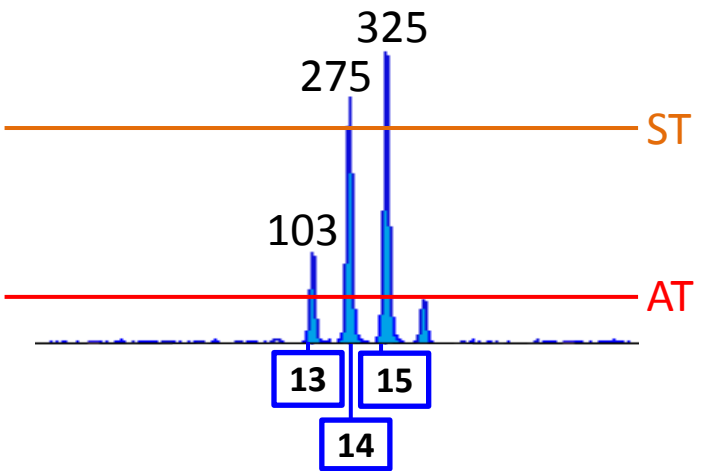
person of
interest (POI)

{13,16}

- 1) What is the probability of obtaining these DNA typing results if the POI is a contributor?

D8S1179

115 155



Role of the forensic scientist



person of interest (POI)

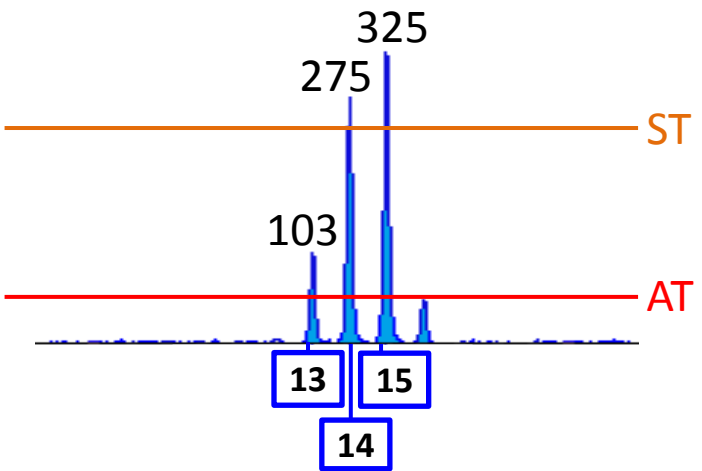
{13,16}

2) What is the probability of obtaining these DNA typing results if the POI is not a contributor?

D8S1179



Role of the forensic scientist



person of interest (POI)

{13,16}



E

Likelihood Ratio (LR)

What is the probability of obtaining these DNA typing results if the POI is a contributor?

$$Pr(E|H_p)$$

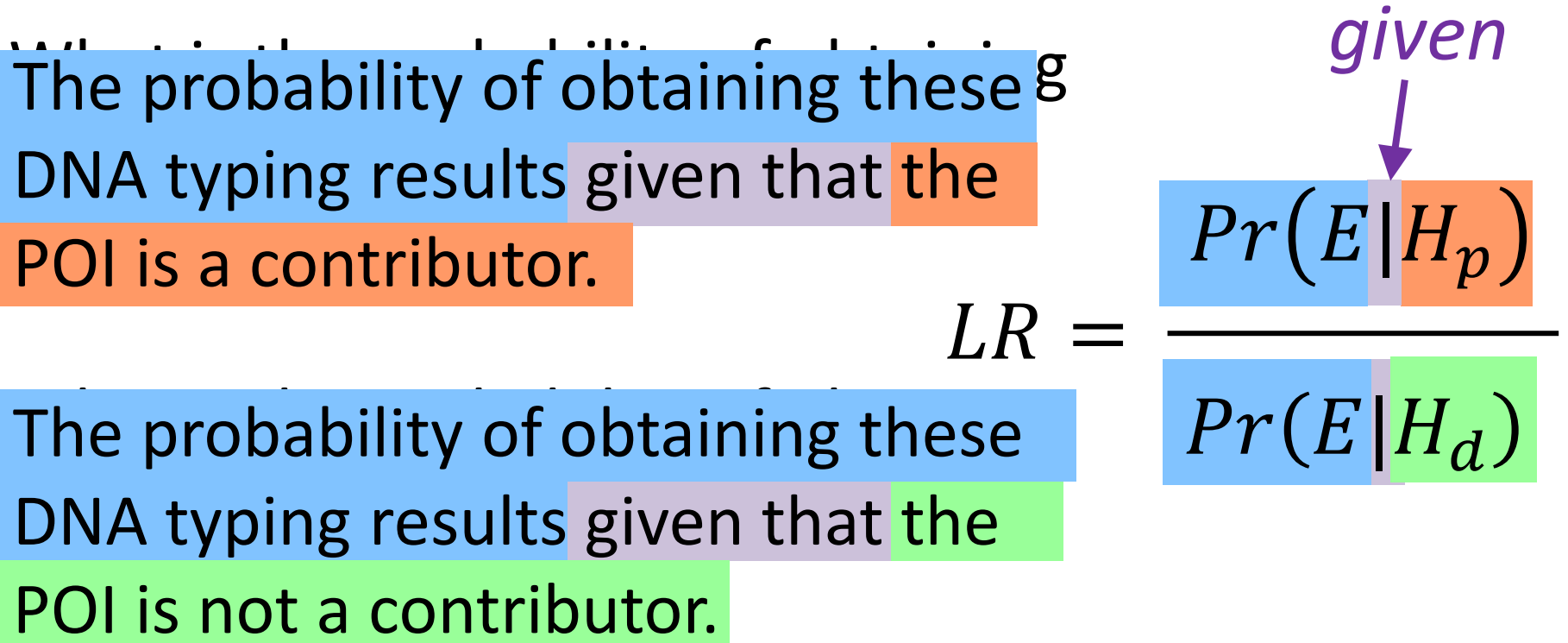
What is the probability of obtaining these DNA typing results if the POI is not a contributor?

$$Pr(E|H_d)$$

Likelihood Ratio (LR)

The probability of obtaining these DNA typing results given that the POI is a contributor.

The probability of obtaining these DNA typing results given that the POI is not a contributor.

$$LR = \frac{Pr(E|H_p)}{Pr(E|H_d)}$$


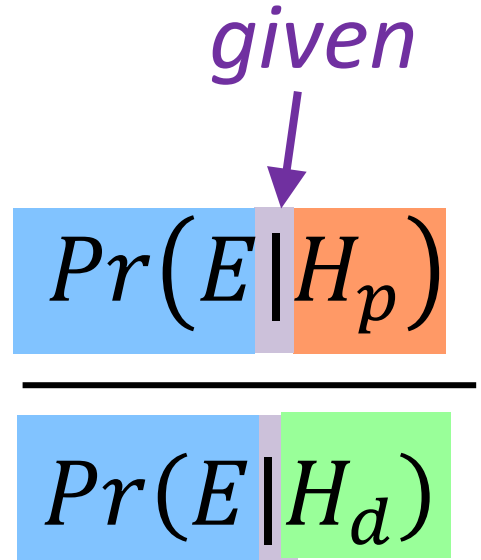
Likelihood Ratio (LR)

~~The probability that the POI is a contributor divided by the probability that the POI is not a contributor.~~

POI is not a contributor.

$LR =$

given

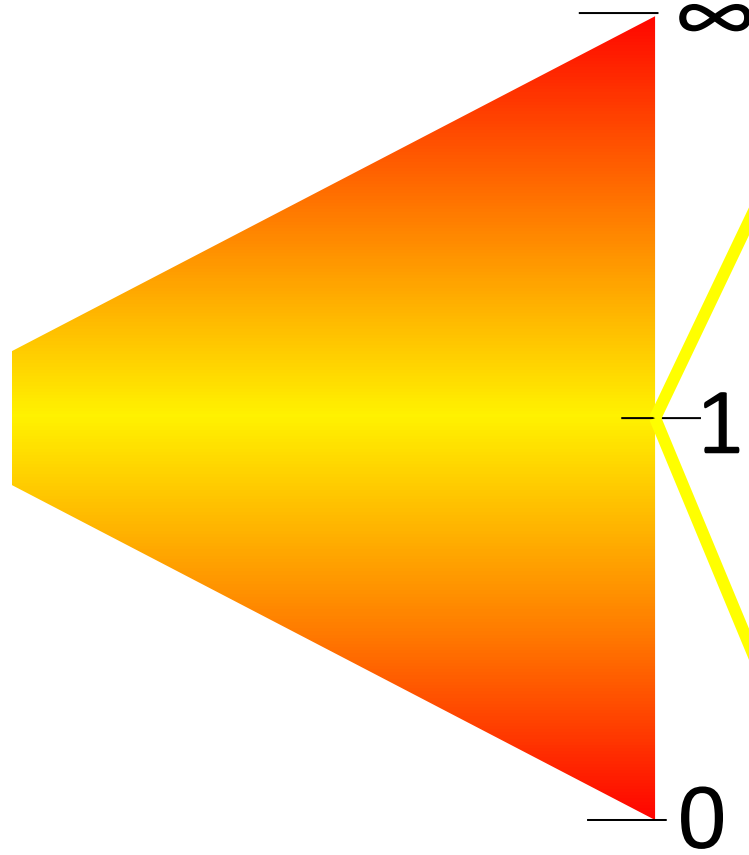


The diagram shows the formula $LR = \frac{Pr(E|H_p)}{Pr(E|H_d)}$. The numerator $Pr(E|H_p)$ is shown in a blue box with an orange box to its right. The denominator $Pr(E|H_d)$ is shown in a blue box with a green box to its right. A purple arrow points from the word "given" above to the vertical bar in the numerator's expression.

$$\frac{Pr(E|H_p)}{Pr(E|H_d)}$$

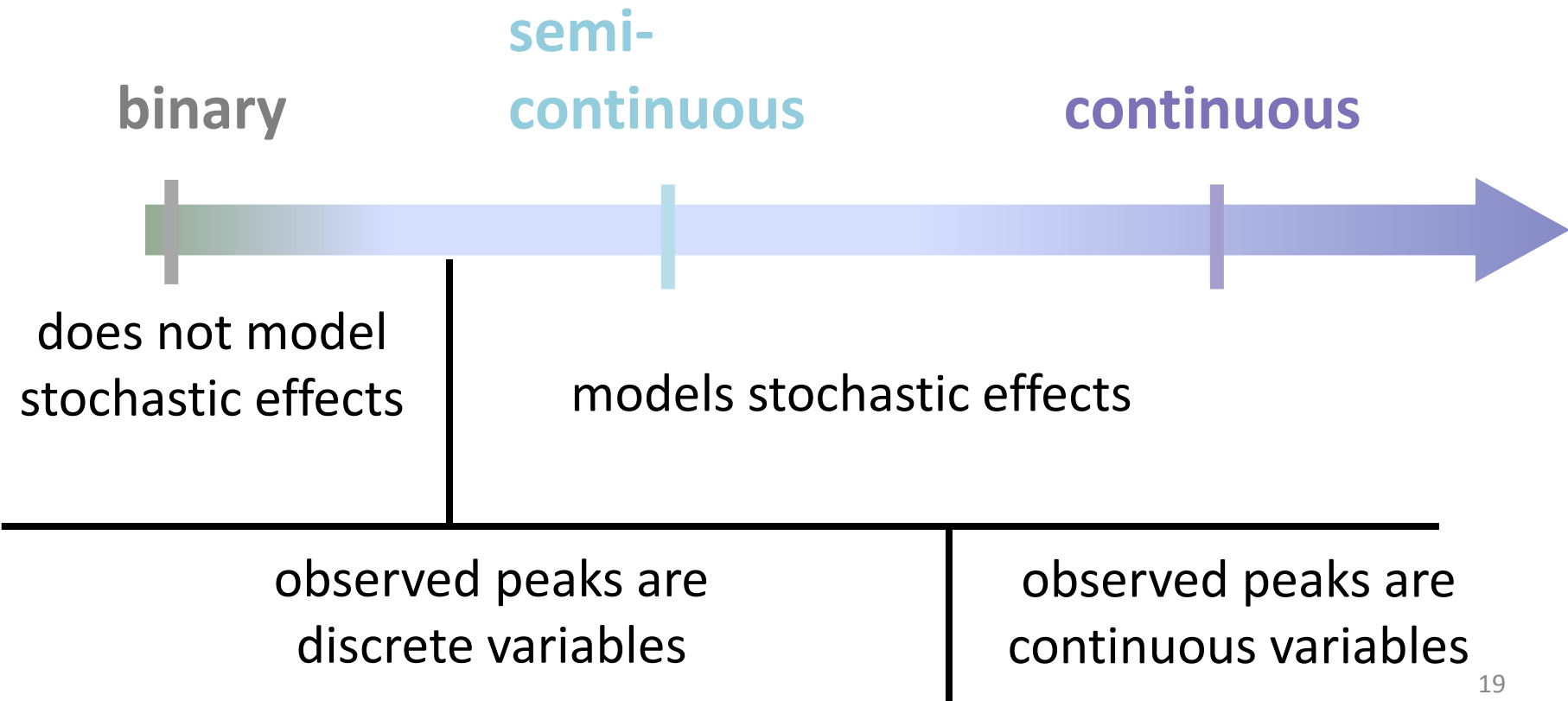
Likelihood Ratio (LR)

$$\frac{\Pr(E | H_p)}{\Pr(E | H_d)}$$



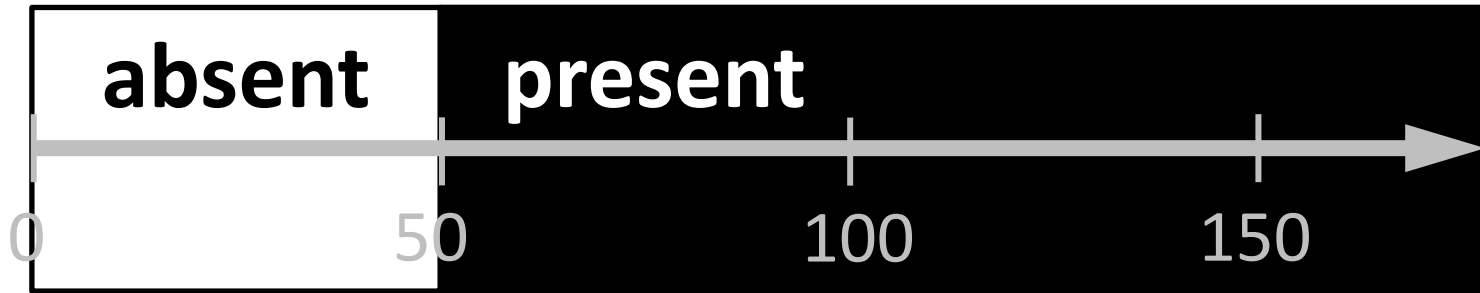
the DNA typing results are just as likely if the POI is a contributor than if the POI is not a contributor

Likelihood Ratio (LR)



Discrete vs. Continuous

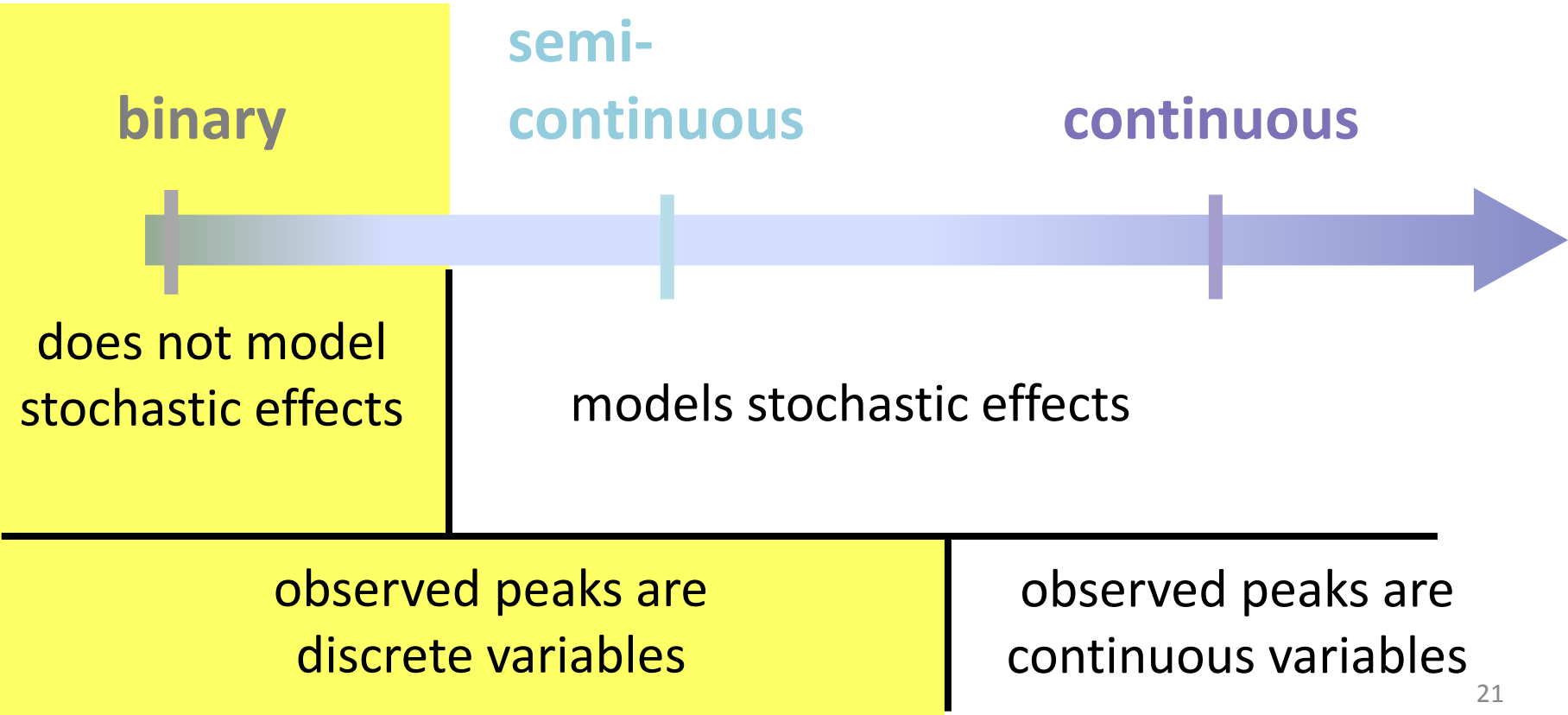
- The observed peaks as a **discrete** variable:



- The observed peaks as a **continuous** variable:



Likelihood Ratio (LR)



D8S1179

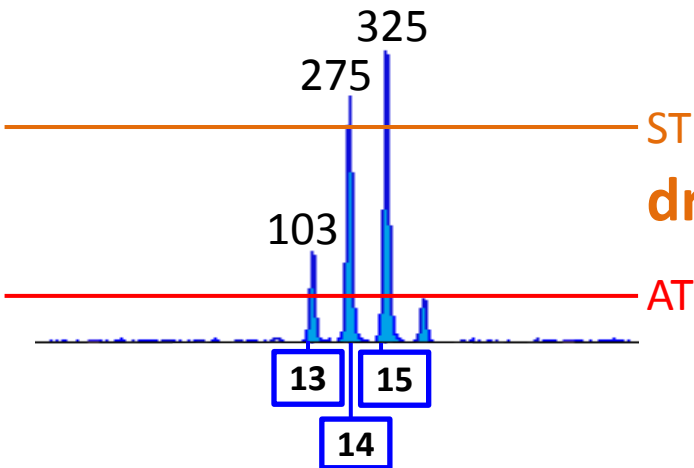


Binary LR



person of
interest (POI)

{13,16}



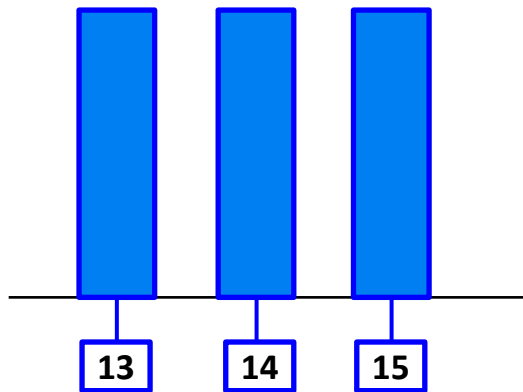
ST

drop-out is possible!

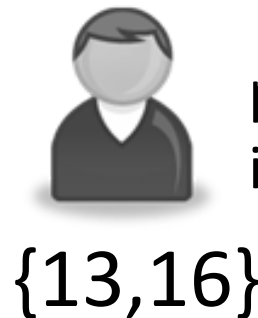
AT

D8S1179

Binary LR



drop-out is possible!



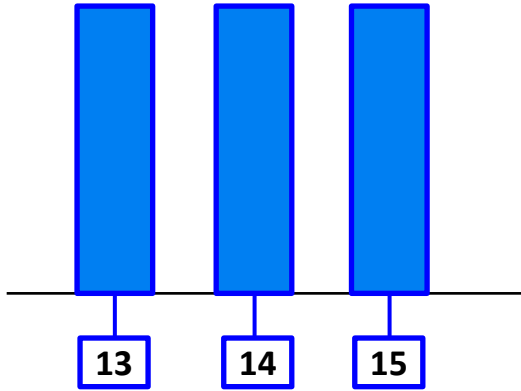
person of
interest (POI)

1) What is the probability of obtaining these DNA typing results if the POI is a contributor?

$$\Pr(E|H_p) = 1 \times 2p_{14}p_{15}$$

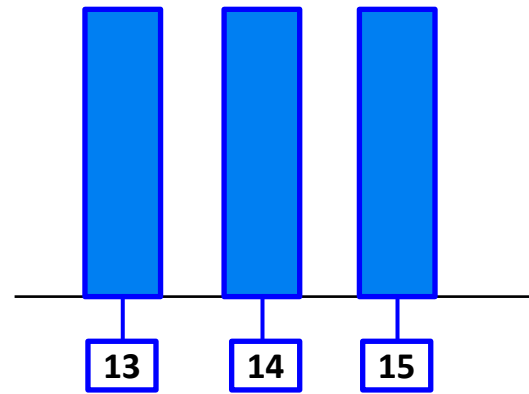
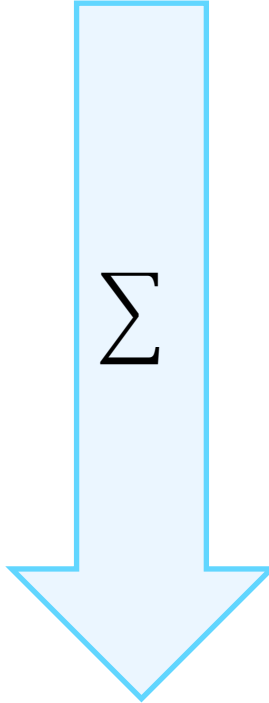
D8S1179

Binary LR



2) What is the probability of obtaining these DNA typing results if the POI **is not** a contributor?

Major	Minor	
13,13	14,15	$p_{13}^2 \times 2p_{14}p_{15}$
14,14	13,15	$p_{14}^2 \times 2p_{13}p_{15}$
15,15	13,14	$p_{15}^2 \times 2p_{13}p_{14}$
14,15	13,13	$2p_{14}p_{15} \times p_{13}^2$
13,15	14,14	$2p_{13}p_{15} \times p_{14}^2$
13,14	15,15	$2p_{13}p_{14} \times p_{15}^2$
13,14	13,15	$2p_{13}p_{14} \times 2p_{13}p_{15}$
13,14	14,15	$2p_{13}p_{14} \times 2p_{14}p_{15}$
13,15	14,15	$2p_{13}p_{15} \times 2p_{14}p_{15}$
13,15	13,14	$2p_{13}p_{15} \times 2p_{13}p_{14}$
14,15	13,14	$2p_{14}p_{15} \times 2p_{13}p_{14}$
14,15	13,15	$2p_{14}p_{15} \times 2p_{13}p_{15}$



$$= 12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15})$$

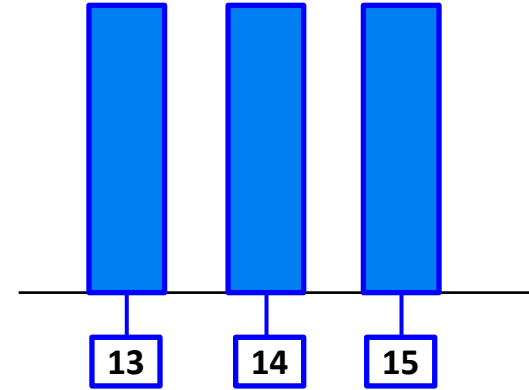
(continued) :

Major	Minor	
13,14	15,Q	$2p_{13}p_{14} \times 2p_{15}p_Q$
13,15	14,Q	$2p_{13}p_{15} \times 2p_{14}p_Q$
14,15	13,Q	$2p_{14}p_{15} \times 2p_{13}p_Q$
15,Q	13,14	$2p_{15}p_Q \times 2p_{13}p_{14}$
14,Q	13,15	$2p_{14}p_Q \times 2p_{13}p_{15}$
13,Q	14,15	$2p_{13}p_Q \times 2p_{14}p_{15}$



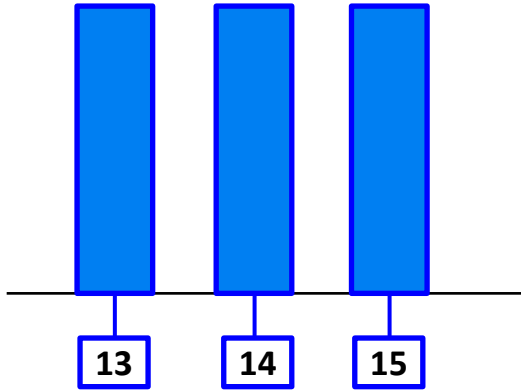
$$\Sigma$$

$$= 24p_{13}p_{14}p_{15}p_Q$$



D8S1179

Binary LR

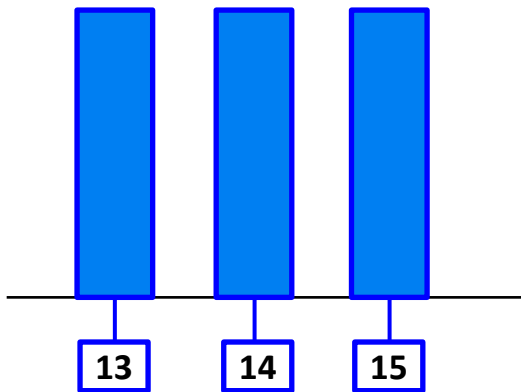


2) What is the probability of obtaining these DNA typing results if the POI **is not** a contributor?

$$\Pr(E|H_d) = 12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q$$

D8S1179

Binary LR

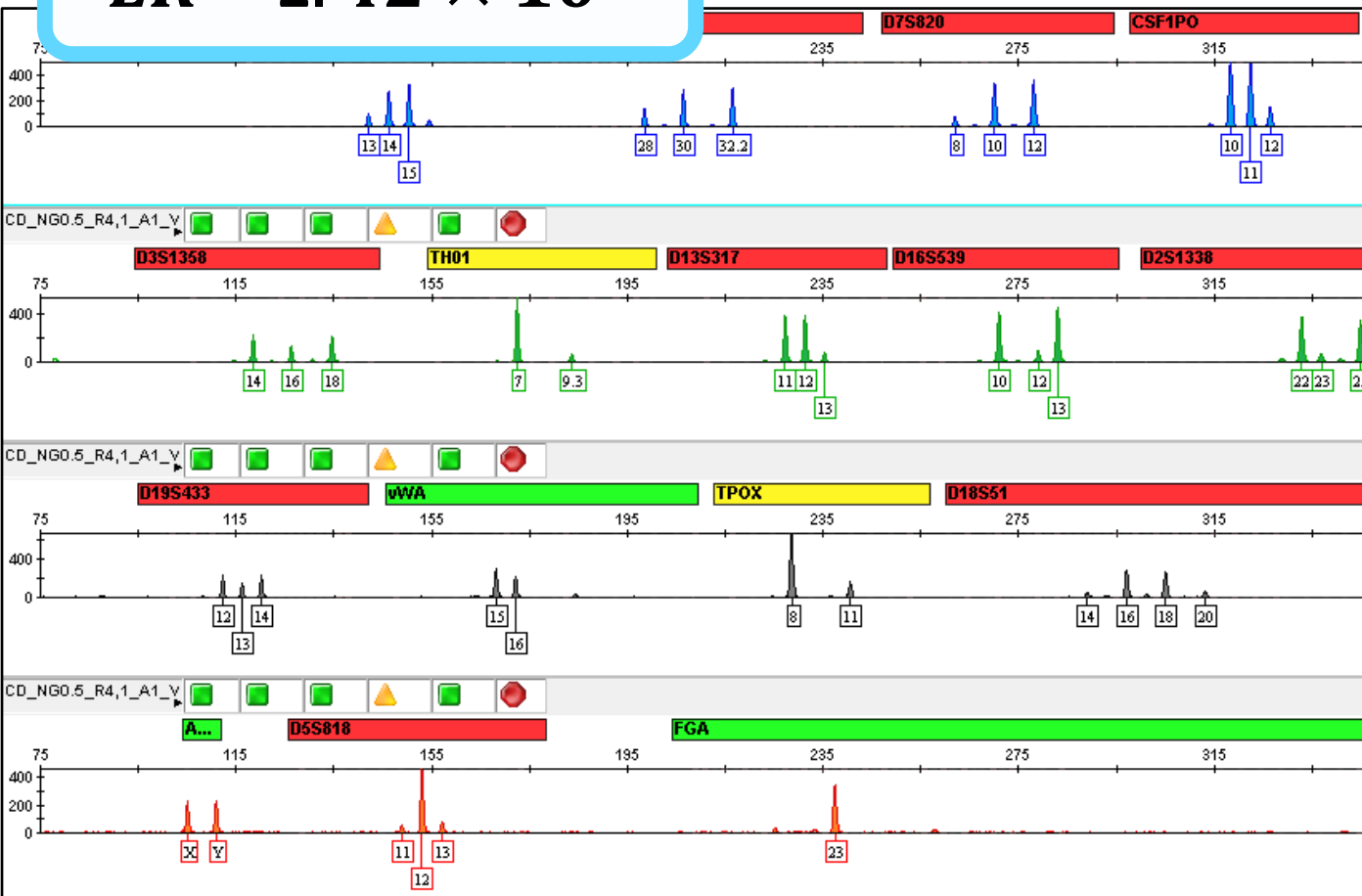


person of
interest (POI)

{13,16}

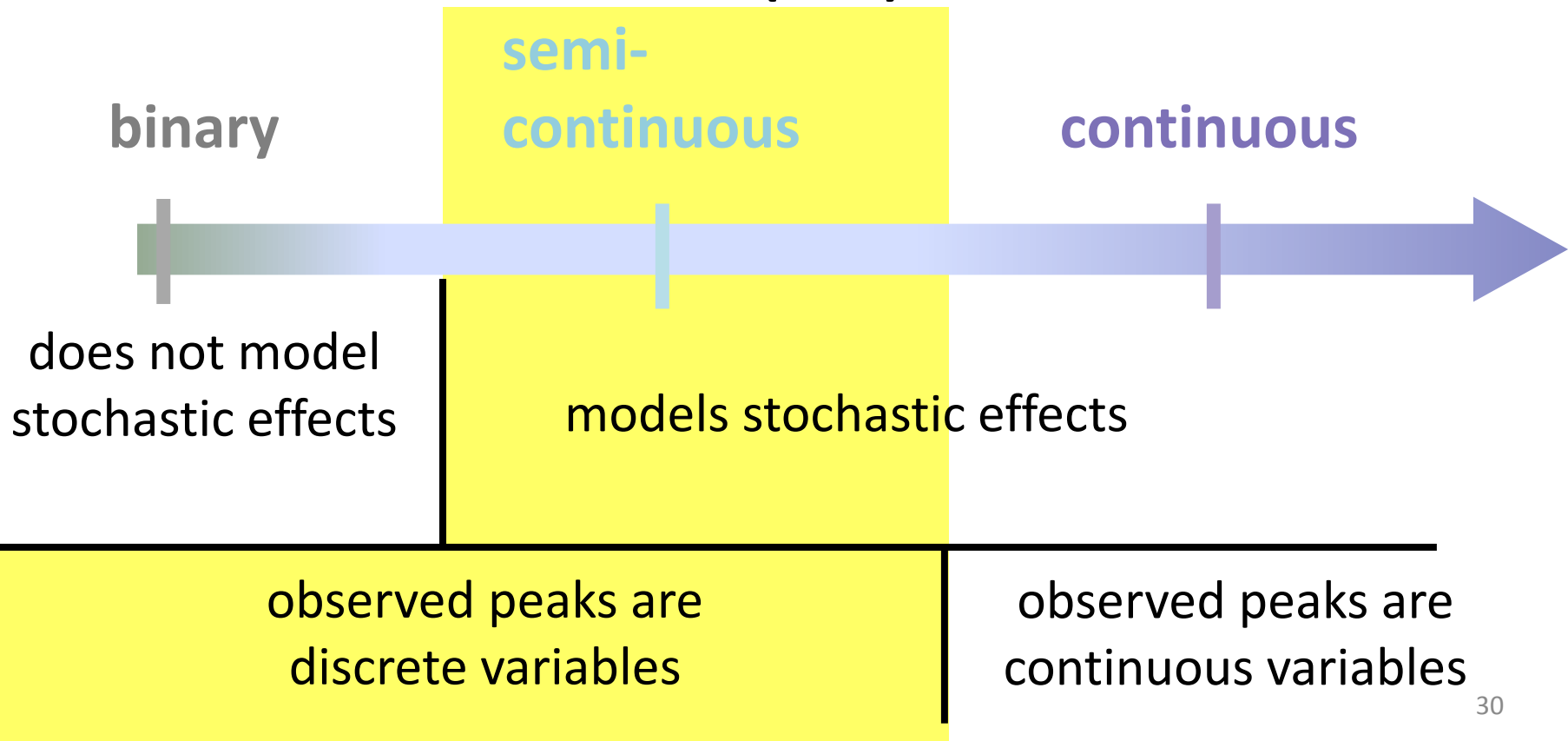
$$\begin{aligned} LR &= \frac{2p_{14}p_{15}}{12p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + 24p_{13}p_{14}p_{15}p_Q} \\ &= \frac{1}{6p_{13}(p_{13} + p_{14} + p_{15}) + 12p_{13}p_Q} = 0.39 \end{aligned}$$

$$LR = 1.42 \times 10^3$$



Locus	POI	Binary LR
D8S1179	13,16	0.39
D21S11	28,28	0.71
D7S820	8,12	4.08
CSF1PO	12,12	0.41
D3S1358	16,16	0.44
TH01	7,9.3	2.12
D3S1358	12,13	3.49
D16S539	12,13	2.32
D2S1338	23,25	8.57
D19S433	13,13	0.51
vWA	15,19	2.05
TPOX	11,11	1.27
D18S51	14,20	28.09
D5S818	11,13	1.66
FGA	20,28	0.76

Likelihood Ratio (LR)



Semi-continuous LR

allele drop-out

heterozygote: $Pr(\text{drop} - \text{out}) = D_H$

$$Pr(\text{not drop} - \text{out}) = 1 - D_H = \overline{D_H}$$

homozygote: $Pr(\text{drop} - \text{out}) = D_{2H}$

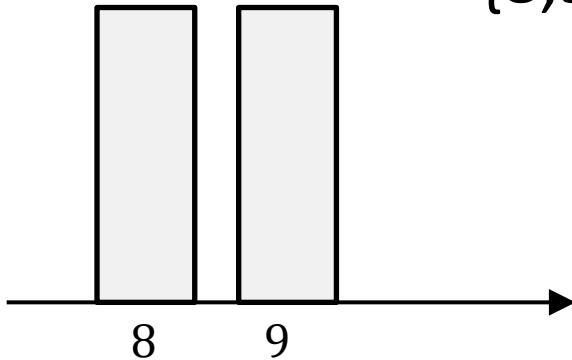
$$Pr(\text{not drop} - \text{out}) = 1 - D_{2H} = \overline{D_{2H}}$$

Semi-continuous LR

Examples:



{8,9}



$$LR = \frac{\overline{D}_H^2}{\overline{D}_H^2 \times 2p_8p_9} = \frac{1}{2p_8p_9}$$

$$Pr(E|H_p) = \overline{D}_H \overline{D}_H \times 1 = \overline{D}_H^2$$

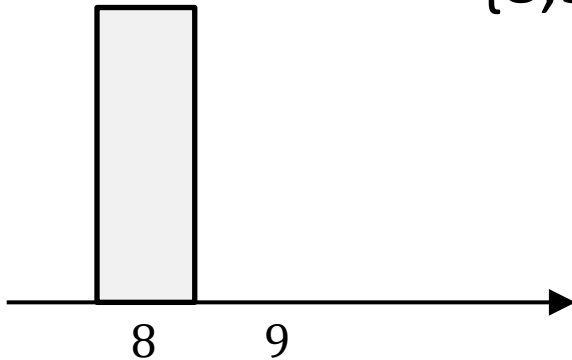
$$Pr(E|H_d) = \overline{D}_H \overline{D}_H \times 2p_8p_9 = \overline{D}_H^2 \times 2p_8p_9$$

Semi-continuous LR

Examples:



{8,9}



Contributor	
8,8	$\overline{D}_{2H} \times p_8^2$
8,Q	$\overline{D}_H D_H \times 2p_8p_Q$

where $p_Q = 1 - p_8$

$$Pr(E|H_p) = \overline{D}_H D_H \times 1 = \overline{D}_H D_H$$

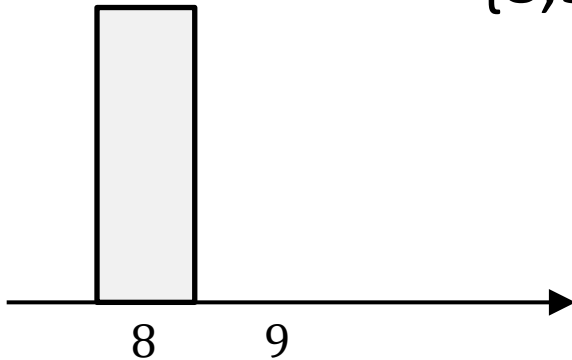
$$Pr(E|H_d) = \overline{D}_{2H} \times p_8^2 + \overline{D}_H D_H \times 2p_8p_Q$$

Semi-continuous LR

Examples:



{8,9}



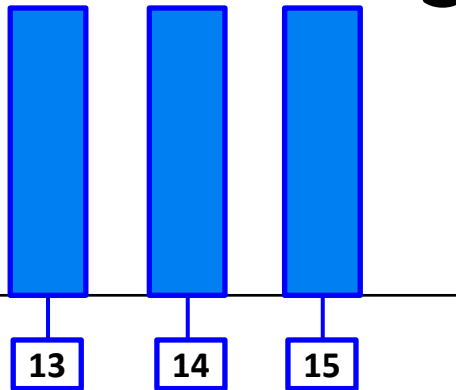
$$LR = \frac{\overline{D}_H D_H}{\overline{D}_{2H} \times p_8^2 + \overline{D}_H D_H \times 2p_8p_Q}$$

$$Pr(E|H_p) = \overline{D}_H D_H \times 1 = \overline{D}_H D_H$$

$$Pr(E|H_d) = \overline{D}_{2H} \times p_8^2 + \overline{D}_H D_H \times 2p_8p_Q$$

D8S1179

Semi-continuous LR



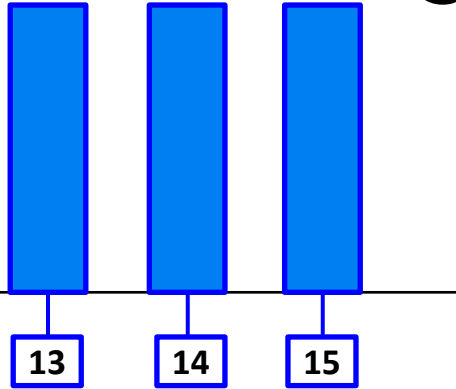
person of
interest (POI)

{13,16}

1) What is the probability of obtaining these DNA typing results if the POI is a contributor?

$$\begin{aligned}\Pr(E|H_p) &= \overline{D_H} D_H \times 1 \times \overline{D_H}^2 \times 2p_{14}p_{15} \\ &= D_H \overline{D_H}^3 \times 2p_{14}p_{15}\end{aligned}$$

D8S1179

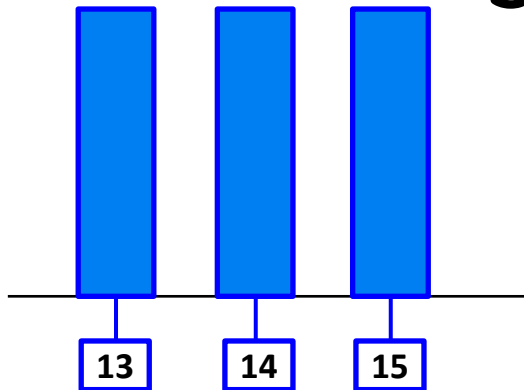


Se

Major	Minor		
13,13	14,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times p_{13}^2 \times 2p_{14}p_{15}$
14,14	13,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times p_{14}^2 \times 2p_{13}p_{15}$
15,15	13,14	$\overline{D_{2H}} \overline{D_H}^2$	$\times p_{15}^2 \times 2p_{13}p_{14}$
14,15	13,13	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{14}p_{15} \times p_{13}^2$
13,15	14,14	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{15} \times p_{14}^2$
13,14	15,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{14} \times p_{15}^2$
13,14	13,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{14} \times 2p_{13}p_{15}$
13,14	14,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{14} \times 2p_{14}p_{15}$
13,15	14,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{15} \times 2p_{14}p_{15}$
13,15	13,14	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{13}p_{15} \times 2p_{13}p_{14}$
14,15	13,14	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{14}p_{15} \times 2p_{13}p_{14}$
14,15	13,15	$\overline{D_{2H}} \overline{D_H}^2$	$\times 2p_{14}p_{15} \times 2p_{13}p_{15}$

2) What is the proba
POI is not a contribu

Semi-continuous LR

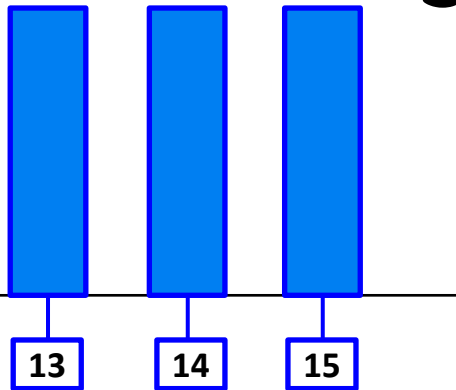


(continued)

Major	Minor		
13,14	15,Q	$\overline{D}_H^3 D_H$	$\times 2p_{13}p_{14} \times 2p_{15} p_Q$
13,15	14,Q	$\overline{D}_H^3 D_H$	$\times 2p_{13}p_{15} \times 2p_{14}p_Q$
14,15	13,Q	$\overline{D}_H^3 D_H$	$\times 2p_{14}p_{15} \times 2p_{13}p_Q$
15,Q	13,14	$\overline{D}_H^3 D_H$	$\times 2p_{15}p_Q \times 2p_{13}p_{14}$
14,Q	13,15	$\overline{D}_H^3 D_H$	$\times 2p_{14}p_Q \times 2p_{13}p_{15}$
13,Q	14,15	$\overline{D}_H^3 D_H$	$\times 2p_{13}p_Q \times 2p_{14}p_{15}$

where $p_Q = 1 - p_{13} - p_{14} - p_{15}$

Semi-continuous LR



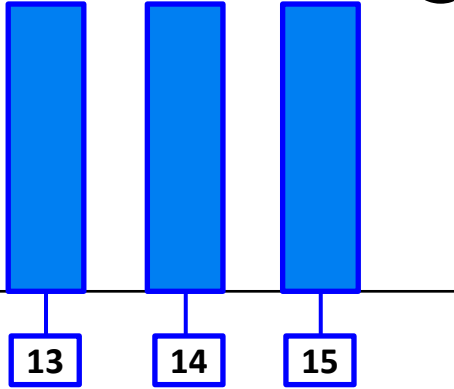
Homs

$$\begin{aligned}
 Pr(E|H_d) = & \overline{D_{2H}} \overline{D_H}^2 \times 4p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) \\
 & + \overline{D_{2H}} \overline{D_H}^2 \times 8p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) \\
 & + \overline{D_H}^3 D_H \times 24p_{13}p_{14}p_{15}p_0
 \end{aligned}$$

Hets with stacking

Hets with drop-out

Semi-continuous LR



$$LR = \frac{D_H \overline{D_H}^3 \times 2p_{14}p_{15}}{\overline{D_{2H}} \overline{D_H}^2 \times 4p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + \overline{D_{2H}} \overline{D_H}^2 \times 8p_{13}p_{14}p_{15}(p_{13} + p_{14} + p_{15}) + \overline{D_H}^3 D_H \times 24p_{13}p_{14}p_{15}p_Q}$$

Major	Minor		
13,13	14,15	0.075	$\times 2p_{13}^2 p_{14} p_{15}$
14,14	13,15	0.075	$\times 2p_{13} p_{14}^2 p_{15}$
15,15	13,14	0.075	$\times 2p_{13} p_{14} p_{15}^2$
14,15	13,13	0.075	$\times 2p_{13}^2 p_{14} p_{15}$
13,15	14,14	0.075	$\times 2p_{13} p_{14}^2 p_{15}$
13,14	15,15	0.075	$\times 2p_{13} p_{14} p_{15}^2$
13,14	13,15	0.075	$\times 4p_{13}^2 p_{14} p_{15}$
13,14	14,15	0.075	$\times 4p_{13} p_{14}^2 p_{15}$
13,15	14,15	0.075	$\times 4p_{13} p_{14} p_{15}^2$
13,15	13,14	0.075	$\times 4p_{13}^2 p_{14} p_{15}$
14,15	13,14	0.075	$\times 4p_{13} p_{14}^2 p_{15}$
14,15	13,15	0.075	$\times 4p_{13} p_{14} p_{15}^2$
13,14	15,Q	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$
13,15	14,Q	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$
14,15	13,Q	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$
15,Q	13,14	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$
14,Q	13,15	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$
13,Q	14,15	0.016	$\times 4p_{13} p_{14} p_{15} p_Q$

$$D_H = 0.716$$

$$D_{2H} = 0.256$$

Semi-Continuous
 $LR = 8.28 \times 10^3$

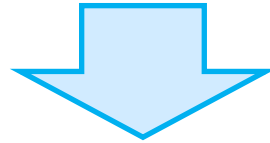
Locus	Binary LR	Semi-Continuous LR
D8S1179	0.39	0.16
D21S11	0.71	1.44
D7S820	4.08	4.75
CSF1PO	0.41	0.50
D3S1358	0.44	0.91
TH01	2.12	2.63
D3S1358	3.49	3.91
D16S539	2.32	2.88
D2S1338	8.57	10.4
D19S433	0.51	0.79
vWA	2.05	1.46
TPOX	1.27	2.06
D18S51	28.09	28.1
D5S818	1.66	1.94
FGA	0.76	0.50

$$D_H = 0.716$$

$$D_{2H} = 0.256$$

Binary

$$LR = 1.42 \times 10^3$$



Semi-Continuous

$$LR = 8.28 \times 10^3$$

Major	Minor		
13,13	14,15	0.075	$\times 2p_{13}^2p_{14}p_{15}$
14,14	13,15	0.075	$\times 2p_{13}p_{14}^2p_{15}$
15,15	13,14	0.075	$\times 2p_{13}p_{14}p_{15}^2$
14,15	13,13	0.075	$\times 2p_{13}^2p_{14}p_{15}$
13,15	14,14	0.075	$\times 2p_{13}p_{14}^2p_{15}$
13,14	15,15	0.075	
13,14	13,15	0.075	
13,14	14,15		
13,15	14,15		
13,15	13,14		
14,15	13,14		
14,15	13,15	0.075	
13,14	15,Q	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$
13,15	14,Q	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$
14,15	13,Q	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$
15,Q	13,14	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$
14,Q	13,15	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$
13,Q	14,15	0.016	$\times 4p_{13}p_{14}p_{15}p_Q$

$$D_H = 0.716$$

$$D_{2H} = 0.256$$

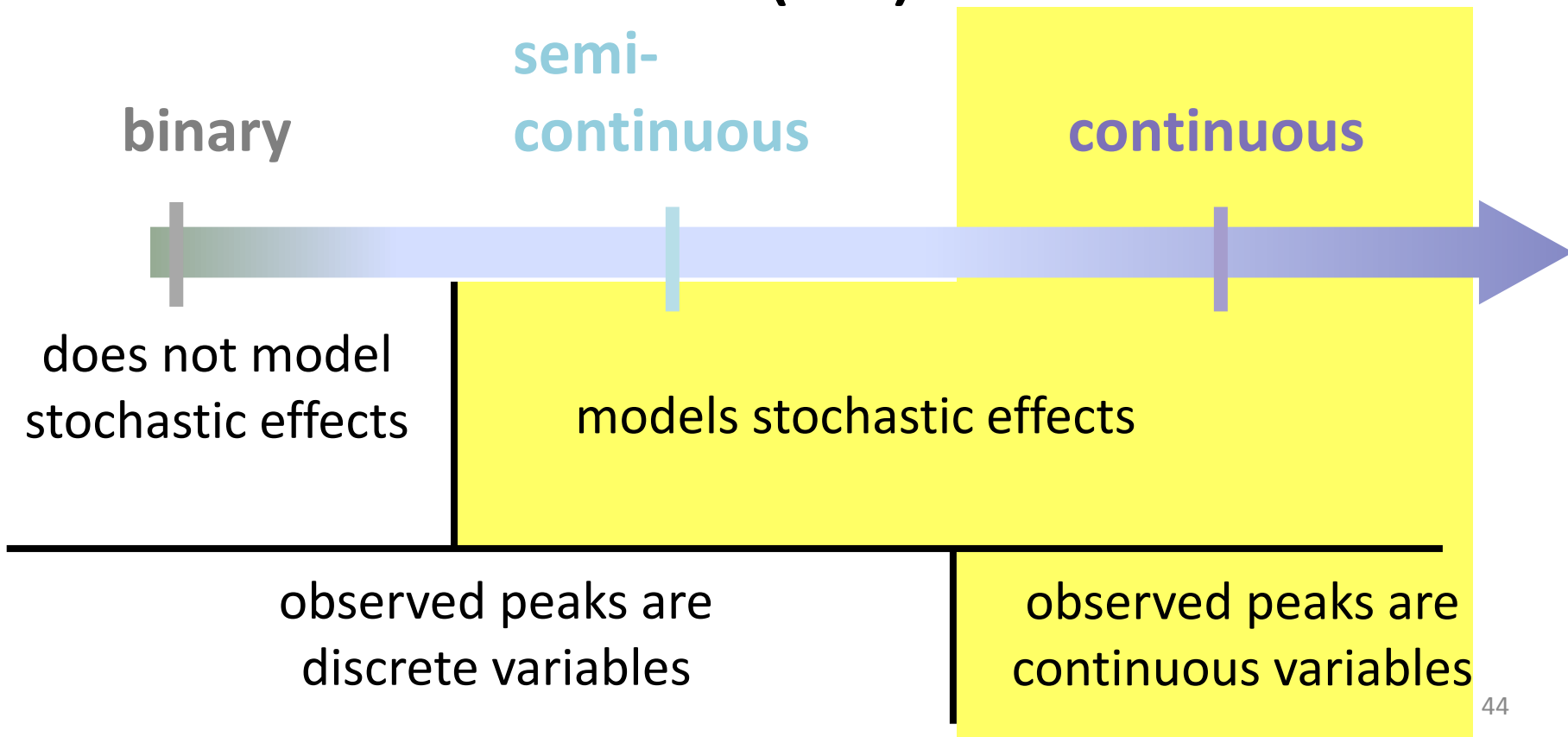
weight

genotype
match
probability

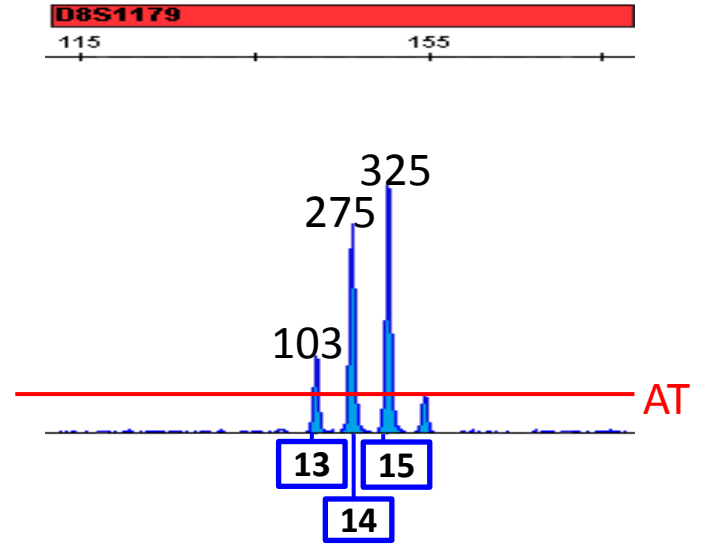
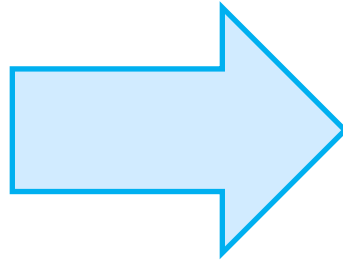
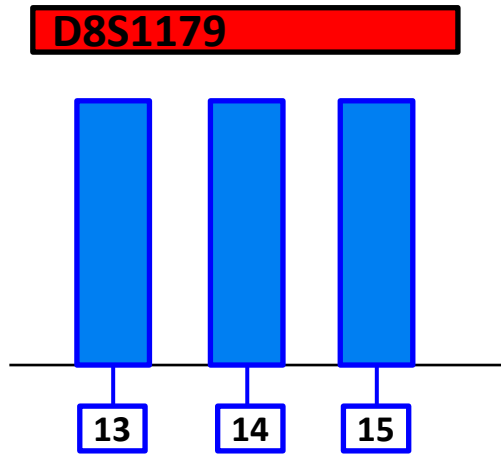
Semi-continuous LR

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \textit{weight} \times \textit{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \textit{weight} \times \textit{genotype match prob.}}$$

Likelihood Ratio (LR)



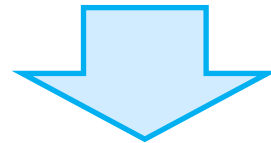
From Semi-continuous to Continuous



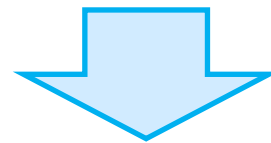
$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

Locus	Binary LR	Semi-Continuous LR	Continuous LR
D8S1179	0.39	0.16	0.09
D21S11	0.71	1.44	38.9
D7S820	4.08	4.75	15.7
CSF1PO	0.41	0.50	7.39
D3S1358	0.44	0.91	15.4
TH01	2.12	2.63	5.88
D3S1358	3.49	3.91	8.87
D16S539	2.32	2.88	10.2
D2S1338	8.57	10.4	11.2
D19S433	0.51	0.79	13.9
vWA	2.05	1.46	3.19
TPOX	1.27	2.06	16.7
D18S51	28.09	28.1	169
D5S818	1.66	1.94	9.07
FGA	0.76	0.50	1

Binary
 $LR = 1.42 \times 10^3$

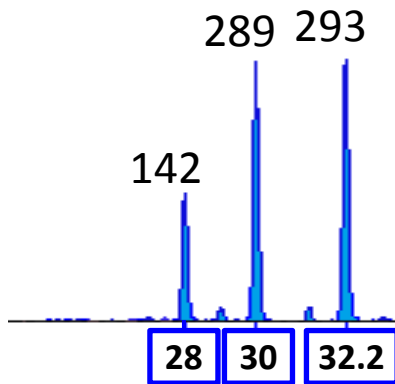
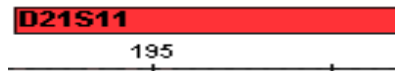


Semi-Continuous
 $LR = 8.28 \times 10^3$



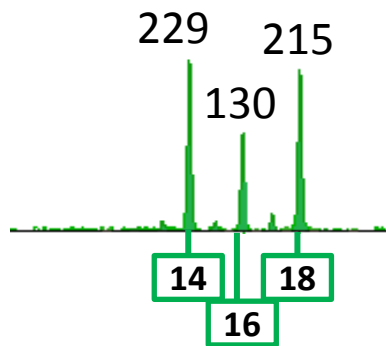
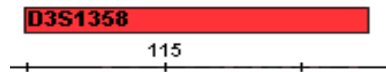
Continuous
 $LR = 4.36 \times 10^{13}$

Locus	Binary	Semi-Continuous	Continuous
D8S1179	0.39	0.16	0.09
D21S11	0.71	1.44	38.9
D7S820	4.08	4.75	15.7
CSF1PO	0.41	0.50	7.39
D3S1358	0.44	0.91	15.4
TH01	2.12	2.63	5.88
D3S1358	3.49	3.91	8.87
D16S539	2.32	2.88	10.2
D2S1338	8.57	10.4	11.2
D19S433	0.51	0.79	13.9
vWA	2.05	1.46	3.19
TPOX	1.27	2.06	16.7
D18S51	28.09	28.1	169
D5S818	1.66	1.94	9.07
FGA	0.76	0.50	1



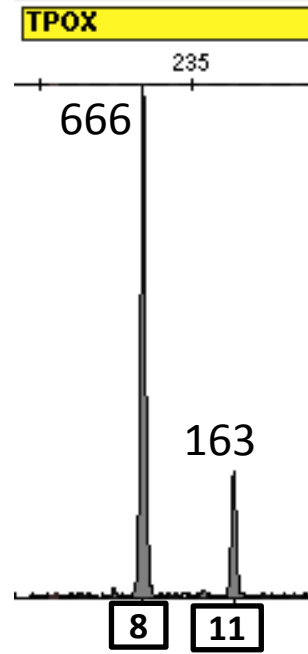
{28,28}

Locus	Binary	Semi-Continuous	Continuous
D8S1179	0.39	0.16	0.09
D21S11	0.71	1.44	38.9
D7S820	4.08	4.75	15.7
CSF1PO	0.41	0.50	7.39
D3S1358	0.44	0.91	15.4
TH01	2.12	2.63	5.88
D3S1358	3.49	3.91	8.87
D16S539	2.32	2.88	10.2
D2S1338	8.57	10.4	11.2
D19S433	0.51	0.79	13.9
vWA	2.05	1.46	3.19
TPOX	1.27	2.06	16.7
D18S51	28.09	28.1	169
D5S818	1.66	1.94	9.07
FGA	0.76	0.50	1



{16,16}

Locus	Binary	Semi-Continuous	Continuous
D8S1179	0.39	0.16	0.09
D21S11	0.71	1.44	38.9
D7S820	4.08	4.75	15.7
CSF1PO	0.41	0.50	7.39
D3S1358	0.44	0.91	15.4
TH01	2.12	2.63	5.88
D3S1358	3.49	3.91	8.87
D16S539	2.32	2.88	10.2
D2S1338	8.57	10.4	11.2
D19S433	0.51	0.79	13.9
vWA	2.05	1.46	3.19
TPOX	1.27	2.06	16.7
D18S51	28.09	28.1	169
D5S818	1.66	1.94	9.07
FGA	0.76	0.50	1



Continuous LR

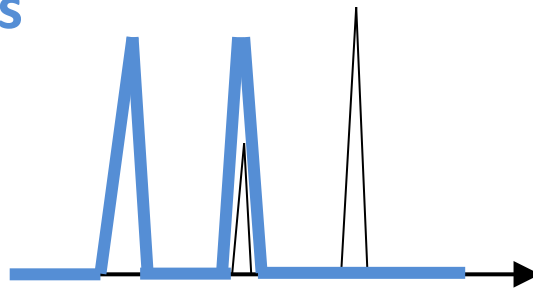
$$LR = \frac{\sum_{genotype\ sets|H_p} \textit{weight} \times \textit{genotype\ match\ prob.}}{\sum_{genotype\ sets|H_d} \textit{weight} \times \textit{genotype\ match\ prob.}}$$

We can obtain the weights through simulations that attempt to reproduce the observed peak heights by varying the genotype set and model parameters.

Continuous LR

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

simulated peak heights

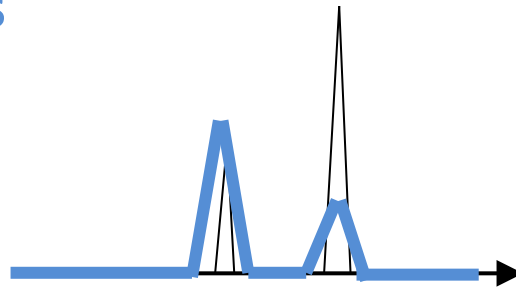


observed peak heights

Continuous LR

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

simulated peak heights

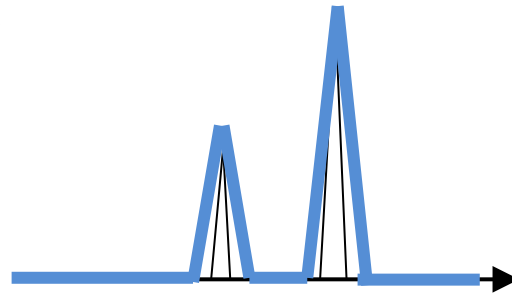


observed peak heights

Continuous LR

$$LR = \frac{\sum_{\text{genotype sets}|H_p} \text{weight} \times \text{genotype match prob.}}{\sum_{\text{genotype sets}|H_d} \text{weight} \times \text{genotype match prob.}}$$

simulated peak heights



observed peak heights

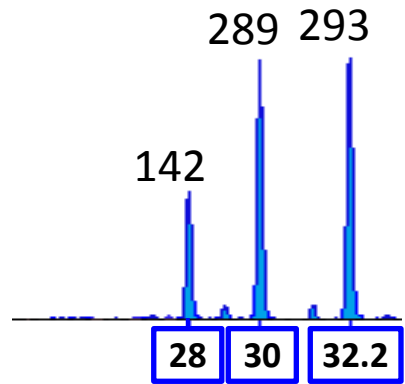
Continuous LR

$$LR = \frac{\sum_{genotype\ sets|H_p} \textit{weight} \times \textit{genotype\ match\ prob.}}{\sum_{genotype\ sets|H_d} \textit{weight} \times \textit{genotype\ match\ prob.}}$$

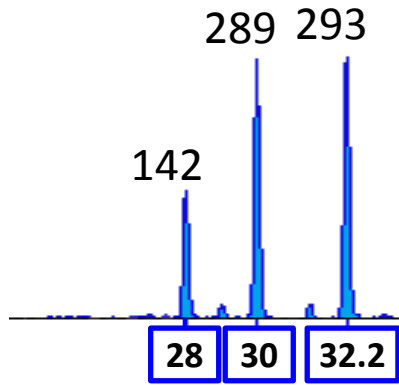
The better the genotype set and the parameters explain the observed peak heights, the greater the weight for that genotype set.

Major	Minor	Binary Weight	Semi-continuous Weight	Continuous Weight
28,28	30,32.2	0.056	0.075	0
30,30	28,32.2	0.056	0.075	0
32.2,32.2	28,30	0.056	0.075	0
30,32.2	28,28	0.056	0.075	0.992
28,32.2	30,30	0.056	0.075	0
28,30	32.2,32.2	0.056	0.075	0
28,30	28,32.2	0.056	0.075	0
28,30	30,32.2	0.056	0.075	0
28,32.2	30,32.2	0.056	0.075	0
28,32.2	28,30	0.056	0.075	0
30,32.2	28,30	0.056	0.075	0.003
30,32.2	28,32.2	0.056	0.075	0.004
28,30	32.2,Q	0.056	0.016	0
28,32.2	30,Q	0.056	0.016	0
30,32.2	28,Q	0.056	0.016	0.001
32.2,Q	28,30	0.056	0.016	0
30,Q	28,32.2	0.056	0.016	0
28,Q	30,32.2	0.056	0.016	0

D21S11



{28,28}



Continuous LR

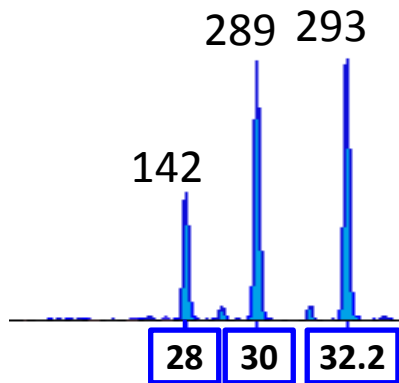


person of
interest (POI)

{28,28}

Major	Minor	Weight	Genotype Match Prob.
30,32.2	28,28	0.992	$\times 2p_{30}p_{32.2}$

$$Pr(E|H_p) = 0.992 \times 2p_{30}p_{32.2}$$

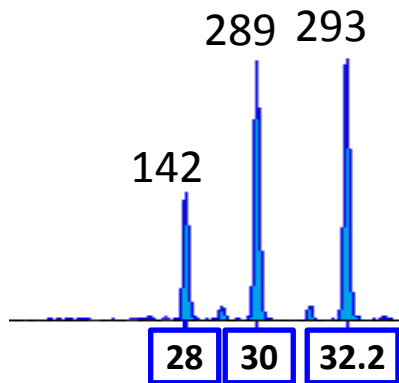


Continuous LR

Major	Minor	Weight	Genotype Match Prob.
30,32.2	28,28	0.992	$\times 2p_{30}p_{32.2} \times p_{28}^2$
30,32.2	28,32.2	0.004	$\times 2p_{30}p_{32.2} \times 2p_{28}p_{32.2}$
30,32.2	28,30	0.003	$\times 2p_{30}p_{32.2} \times 2p_{28}p_{30}$
30,32.2	28,Q	0.001	$\times 2p_{30}p_{32.2} \times 2p_{28}p_Q$

where $p_Q = 1 - p_{28} - p_{30} - p_{32.2}$

$$Pr(E|H_d) = 0.992 \times 2p_{30}p_{32.2}p_{28}^2 + 0.004 \times 4p_{28}p_{30}p_{32.2}^2 \\ + 0.003 \times 4p_{28}p_{30}^2p_{32.2} + 0.001 \times 4p_{28}p_{30}p_{32.2}p_Q$$



Continuous LR

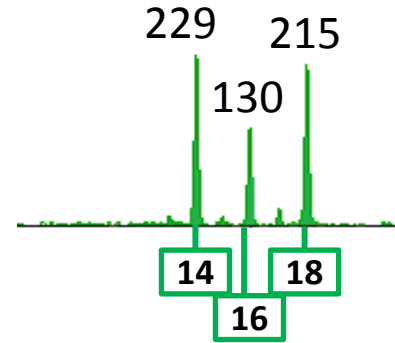
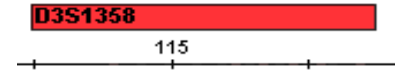


person of
interest (POI)

{28,28}

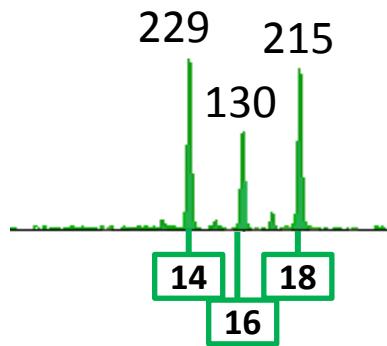
$$\begin{aligned}
 \text{LR} &= \frac{0.992 \times 2p_{30}p_{32.2}}{0.992 \times 2p_{30}p_{32.2}p_{28}^2 + 0.004 \times 4p_{28}p_{30}p_{32.2}^2} \\
 &\quad + 0.003 \times 4p_{28}p_{30}^2p_{32.2} + 0.001 \times 4p_{28}p_{30}p_{32.2}p_Q \\
 &= 38.9
 \end{aligned}$$

Major	Minor	Binary Weight	Semi-Continuous Weight	Continuous Weight
14,14	16,18	0.056	0.075	0
16,16	14,18	0.056	0.075	0
18,18	14,16	0.056	0.075	0
16,18	14,14	0.056	0.075	0
14,18	16,16	0.056	0.075	0.991
14,16	18,18	0.056	0.075	0
14,16	14,18	0.056	0.075	0
14,16	16,18	0.056	0.075	0
14,18	16,18	0.056	0.075	0.003
14,18	14,16	0.056	0.075	0.003
16,18	14,16	0.056	0.075	0
16,18	14,18	0.056	0.075	0
14,16	18,Q	0.056	0.016	0
14,18	16,Q	0.056	0.016	0.002
16,18	14,Q	0.056	0.016	0
18,Q	14,16	0.056	0.016	0
16,Q	14,18	0.056	0.016	0
14,Q	16,18	0.056	0.016	0



{16,16}

Continuous LR



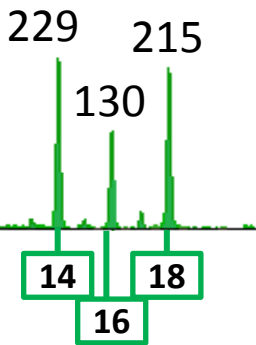
person of
interest (POI)

{16,16}

Major	Minor	Weight	Genotype Match Prob.
14,18	16,16	0.991	$\times 2p_{14}p_{18}$

$$\Pr(E|H_p) = 0.991 \times 2p_{14}p_{18}$$

Continuous LR

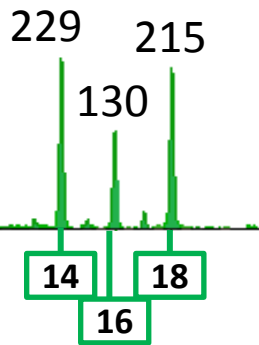


Major	Minor	Weight	Genotype Match Prob.
14,18	16,16	0.991	$\times 2p_{14}p_{18} \times p_{16}^2$
14,18	14,16	0.003	$\times 2p_{14}p_{18} \times 2p_{14}p_{16}$
14,18	16,18	0.003	$\times 2p_{14}p_{18} \times 2p_{16}p_{18}$
14,18	16,Q	0.002	$\times 2p_{14}p_{18} \times 2p_{16} p_Q$

where $p_Q = 1 - p_{14} - p_{16} - p_{18}$

$$Pr(E|H_d) = 0.991 \times 2p_{14}p_{18}p_{16}^2 + 0.003 \times 4p_{14}^2p_{16}p_{18} + 0.003 \times 4p_{14}p_{16}p_{18}^2 + 0.002 \times 4p_{14}p_{16}p_{18}p_Q$$

Continuous LR



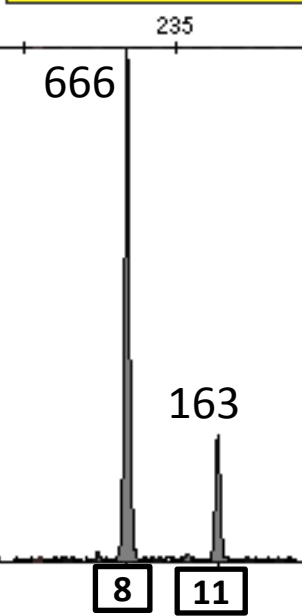
person of
interest (POI)

{16,16}

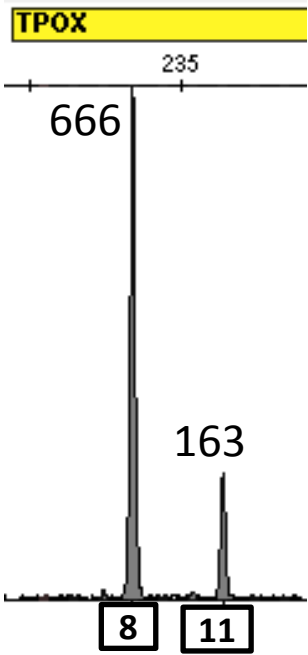
$$\begin{aligned}
 LR &= \frac{0.991 \times 2p_{14}p_{18}}{0.991 \times 2p_{14}p_{18}p_{16}^2 + 0.003 \times 4p_{14}^2p_{16}p_{18} \\
 &\quad + 0.003 \times 4p_{14}p_{16}p_{18}^2 + 0.002 \times 4p_{14}p_{16}p_{18}p_Q} \\
 &= 15.4
 \end{aligned}$$

Major	Minor	Binary Weight	Semi-Continuous Weight	Continuous Weight
8,8	11,11	0.053	0.160	0.997
11,11	8,8	0.053	0.160	0
8,11	8,11	0.053	0.160	0
8,8	8,11	0.053	0.160	0.003
8,11	8,8	0.053	0.160	0
8,11	11,11	0.053	0.160	0
11,11	8,11	0.053	0.160	0
8,11	8,Q	0.053	0.053	0
8,Q	8,11	0.053	0.053	0
8,11	11,Q	0.053	0.053	0
11,Q	8,11	0.053	0.053	0
8,8	11,Q	0.053	0.053	0
11,Q	8,8	0.053	0.053	0
11,11	8,Q	0.053	0.053	0
8,Q	11,11	0.053	0.053	0
8,11	Q,Q	0.053	0.001 or 0.008	0
Q,Q	8,11	0.053	0.001 or 0.008	0
8,Q	11,Q	0.053	0.008	0
11,Q	8,Q	0.053	0.008	0

TPOX



{11,11}



Continuous LR



person of interest (POI)

{11,11}

Major	Minor	Weight	Genotype Match Prob.
8,8	11,11	0.997	$\times p_8^2$

$$Pr(E|H_p) = 0.997 \times p_8^2$$

666

163

8

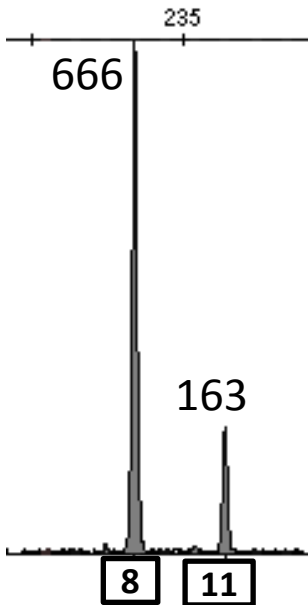
11

Continuous LR

Major	Minor	Weight	Genotype Match Prob.
8,8	11,11	0.997	$\times p_8^2 \times p_{11}^2$
8,8	8,11	0.003	$\times p_8^2 \times 2p_8p_{11}$

where $p_Q = 1 - p_8 - p_{11}$

$$Pr(E|H_d) = 0.997 \times p_8^2 p_{11}^2 + 0.003 \times 2p_8^3 p_{11}$$



Continuous LR



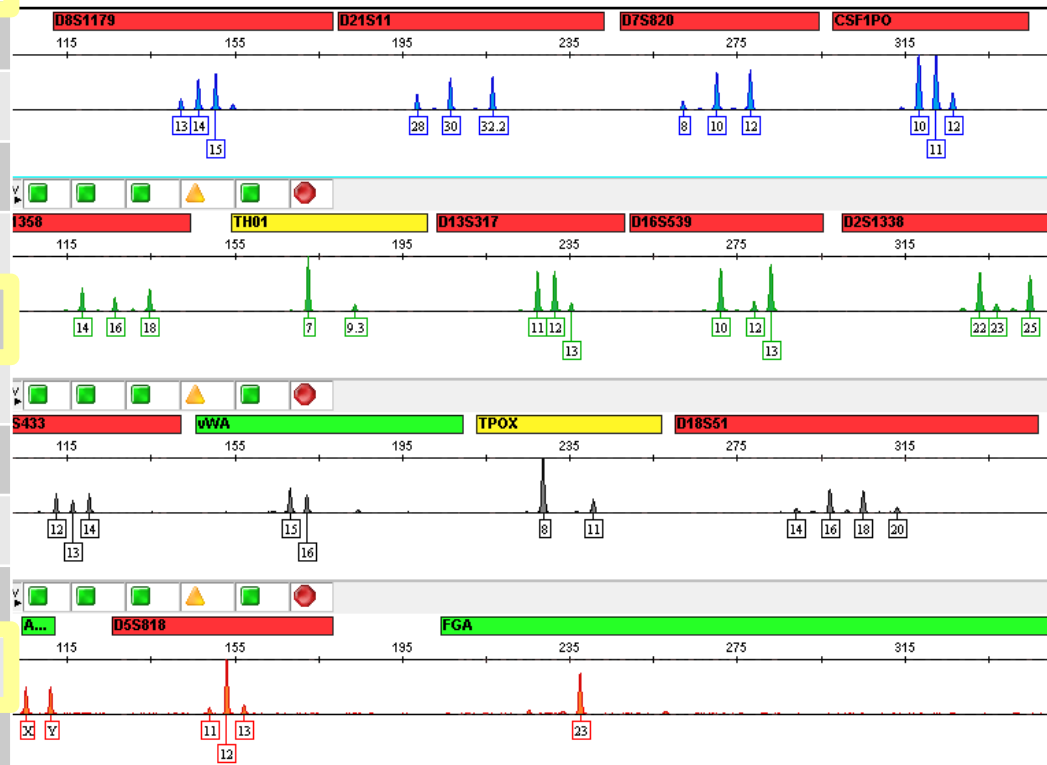
person of
interest (POI)

{11,11}

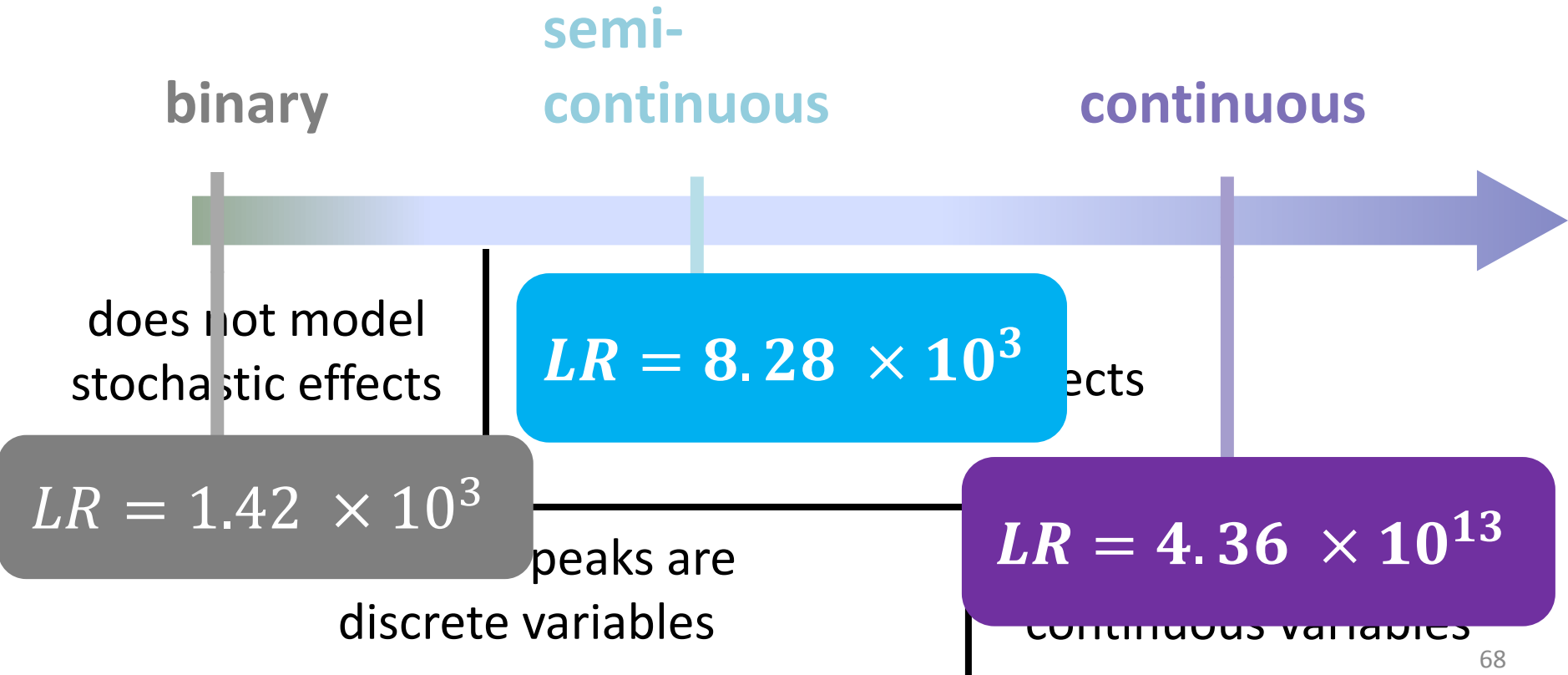
$$LR = \frac{0.997 \times p_8^2}{0.997 \times p_8^2 p_{11}^2 + 0.003 \times 2p_8^3 p_{11}}$$

$$= 16.7$$

Locus	Binary	Semi-Continuous	Continuous
D8S1179	0.39	0.16	0.09
D21S11	0.71	1.44	38.9
D7S820	4.08	4.75	15.7
CSF1PO	0.41	0.50	7.39
D3S1358	0.44	0.91	15.4
TH01	2.12	2.63	5.88
D3S1358	3.49	3.91	8.87
D16S539	2.32	2.88	10.2
D2S1338	8.57	10.4	11.2
D19S433	0.51	0.79	13.9
vWA	2.05	1.46	3.19
TPOX	1.27	2.06	16.7
D18S51	28.09	28.1	169
D5S818	1.66	1.94	9.07
FGA	0.76	0.50	1



Summary



Acknowledgements

John Buckleton

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